QUIZ 5
UNIVERSITY OF MASSACHUSETTS AMHERST
MATH 233 – FALL 2013
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NAME: 4 points

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(1) Compute $\frac{\partial f}{\partial v}$ at the point $(u, v, w) = (1, 0, 2)$ if

(i) $$f(x, y, z) = 3e^{x^2yz}$$

(ii) $$x(u, v, w) = \cos(2u + w)$$

(iii) $$y(u, v, w) = 8v$$

(iv) $$z(u, v, w) = \tan(v) + w^2.$$

ANSWER: Using equations (ii), (iii), and (iv), we see that when $(u, v, w) = (1, 0, 2)$, we have

$$x = \cos(2 + 2) = \cos(4) \qquad y = 0 \qquad z = \tan(0) + 2^2 = 4.$$

The partial derivative $\partial f/\partial v$ is given by the chain rule:

(v) $$\frac{\partial f}{\partial v} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial v} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial v}.$$

We compute the relevant derivatives:

$$\frac{\partial f}{\partial x} = 6xyze^{x^2yz} \qquad \frac{\partial f}{\partial y} = 3x^2ze^{x^2yz} \qquad \frac{\partial f}{\partial z} = 3x^2ye^{x^2yz}$$

$$\frac{\partial f}{\partial x}(\cos(4), 0, 4) = 0 \qquad \frac{\partial f}{\partial y}(\cos(4), 0, 4) = 12\cos^2(4) \qquad \frac{\partial f}{\partial z}(\cos(4), 0, 4) = 0.$$

Since we are evaluating these partial derivatives at $(x, y, z) = (\cos(4), 0, 4)$, it is actually unnecessary to compute $\partial x/\partial v$ and $\partial z/\partial v$, since both are 0 when $y = 0$. But, for the sake of completeness, I will compute them all here.

$$\frac{\partial x}{\partial v} = 0 \qquad \frac{\partial y}{\partial v} = 8 \qquad \frac{\partial z}{\partial v} = \sec^2(v).$$

$$\frac{\partial x}{\partial v}(1, 0, 2) = 0 \qquad \frac{\partial y}{\partial v}(1, 0, 2) = 8 \qquad \frac{\partial z}{\partial v}(1, 0, 2) = \sec^2(0) = 1.$$

Putting all the pieces back into equation (v), we see that

$$\frac{\partial f}{\partial v} = 0 \cdot 0 + \left(12\cos^2(4)\right) \cdot 8 + 0 \cdot 1 = \boxed{96\cos^2(4).}$$

Some of you had confusions when computing the partial derivatives of f, so I'll provide some extra details here.

When computing the partial derivative of f with respect to x, we treat y and z as constants. Note that $3e^{x^2yz}$ is a composition of functions $F(G(x))$ where

$$F(x) = 3e^x \qquad F'(x) = 3e^x$$

$$G(x) = x^2yz \qquad G'(x) = 2xyz.$$

By the chain rule,

$$f_x(x, y, z) = \frac{d}{dx}F(G(x)) = F'(G(x))G'(x) = 3e^{x^2yz}(2xyz) = 6xyze^{x^2yz}.$$