QUIZ 6
UNIVERSITY OF MASSACHUSETTS AMHERST
MATH 233 – FALL 2013
NOVEMBER 13, 2013

NAME: 4 points per problem

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(1) Evaluate the following integral. $\int_0^1 \int_0^{\pi/2} x\cos(y)\,dy\,dx$

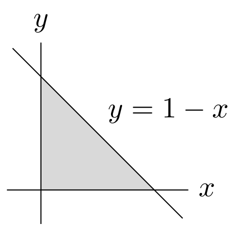
ANSWER:

$$\int_0^1 \int_0^{\pi/2} x\cos(y)\,dy\,dx = \int_0^1 x\sin(y)\Big|_0^{\pi/2}\,dx = \int_0^1 x\,dx = \frac{x^2}{2}\Big|_0^1 = \frac{1}{2}.$$

Alternatively,

$$\int_0^1 \int_0^{\pi/2} x\cos(y)\,dy\,dx = \int_0^1 x\,dx \int_0^{\pi/2} \cos(y)\,dy = \left(\frac{x^2}{2}\Big|_0^1\right)\left(\sin(y)\Big|_0^{\pi/2}\right) = \frac{1}{2}.$$

(2) Set up and evaluate an integral to find the volume under the function $f(x,y) = xy$ over the region in the plane bounded by $y = 0$, $x = 0$, and $y = 1 - x$.



ANSWER:

$$\int_0^1 \int_0^{1-x} xy\,dy\,dx = \int_0^1 \frac{xy^2}{2}\Big|_0^{x-1}\,dx = \frac{1}{2}\int_0^1 x - 2x^2 + x^3\,dx$$
$$= \frac{1}{2}\left[\frac{x^2}{2} - \frac{2x^3}{3} + \frac{x^4}{4}\right]_0^1 = \frac{1}{2}\left(\frac{1}{2} - \frac{2}{3} + \frac{1}{4}\right) = \frac{1}{8}.$$

Alternatively, the volume is given by

$$\int_0^1 \int_0^{1-y} xy\,dx\,dy = \int_0^1 \frac{x^2y}{2}\Big|_0^{1-y}\,dy = \frac{1}{2}\int_0^1 y - 2y^2 + y^3\,dy$$
$$= \frac{1}{2}\left[\frac{y^2}{2} - \frac{2y^3}{3} + \frac{y^4}{4}\right]_0^1 = \frac{1}{2}\left(\frac{1}{2} - \frac{2}{3} + \frac{1}{4}\right) = \frac{1}{8}.$$