Section 1.1:
Sets

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$$\{\text{a, b, c, \ldots, z}\}$$

Natural numbers	$\mathbb{N} = \{1, 2, 3, 4, \ldots\}$
Integers	$\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, \ldots\}$

Notation

We write $a \in A$ to denote that a is an element of the set A. We write $a \notin A$ if a is not an element of A.

The symbols $\in$ means "is an element of the set".
(Think "E" for "element".)

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If $b \in Y$...	"If b is an element of Y..."
Let $x \in A$...	"Let x be an element of A..."
Suppose $n \notin \mathbb{Z}$...	"Suppose n is not an integer..."

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Example:

$$A = \{\{1\}, \{2, 3\}, 3\}$$

$$\{1\} \in A \qquad \{2, 3\} \in A \qquad 3 \in A$$

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$\{1\} \in A$	$\{2,3\} \in A$	$3 \in A$
$1 \notin A$	$2 \notin A$	$\{3\} \notin A$
$1 \in \{1\}$	$2 \in \{2,3\}$	

Definition

Two sets are *equal* if they contain exactly the same elements.

Examples:

$$\begin{aligned} \{\mathrm{red}, \mathrm{green}, \mathrm{blue}\} &\neq \{\mathrm{r}, \mathrm{g}, \mathrm{b}\} \\ \{\{1\}, 2, 3\} &\neq \{1, 2, 3\} \\ \{1, 2, 3\} &= \{1, 2, 3\} \end{aligned}$$

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Order does not matter. Repeated elements are superfluous.

Definition

The *cardinality* (or *size*) of a set is the number of elements in the set. A set is *infinite* if it contains infinitely many elements; otherwise the set is *finite*.

Examples:

Finite of size 3:	$\{\mathrm{r}, \mathrm{g}, \mathrm{b}\}, \{1, 2, 3\}, \{1, \mathrm{cat}, \psi\}$
Infinite:	$\mathbb{N}, \mathbb{Z}$

Notation

If A is a finite set, the cardinality of A is denoted by

$$|A| \quad \text{or} \quad \#A.$$

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$$|\{1,2,3\}| = \#\{1,2,3\} = 3$$

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$$\#\{3,2,1,2,3\} = \#\{1,2,3\} = 3$$

Definition

The *empty set* is the set that contains zero elements.

Notation

The empty set is denoted by $\{\}$ or $\varnothing$.

$$\#\{\} = |\varnothing| = 0$$

$$\{3, 5, 7, 11, \ldots\}$$

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Twin primes	$\{3, 5, 7, 11, 13, 17, 19, 29, 31, \dots\}$
Integer part of $\frac{3^n}{2^n}$ $(n \geq 3)$	$\{3, 5, 7, 11, 17, 25, 38, 57, \dots\}$

Notation

Set-builder notation is an explicit description of a set in terms of rules. Set-build notation has the following presentation:

$$\{\text{expression} : \text{rules}\}.$$

Example:

$$\begin{aligned}\text{The set of even integers} &= \{n \in \mathbb{Z} : n \text{ is even}\} \\ &= \{n : n = 2k, k \in \mathbb{Z}\} \\ &= \{2n : n \in \mathbb{Z}\}.\end{aligned}$$

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Example: *Rational numbers*:

$$\mathbb{Q} = \left\{ \frac{a}{b} : a \in \mathbb{Z}, b \in \mathbb{Z}, b \neq 0 \right\}.$$

Exercises.

1. Write the following set by listing its elements between braces: $\{5x - 1 : x \in \mathbb{Z}\}$.
2. Write the following set in set-builder notation: $\{2, 4, 8, 16, 32, 64, \ldots\}$.
3. Evaluate the following: $|\{\{1, \{2, \{3, 4\}\}, \varnothing\}|$.
4. Sketch the following sets of points in the x, y-plane: $\{(x, y) : x \in [1, 2], y \in [1, 2]\}$.

Exercises.

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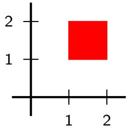
4. Sketch the following sets of points in the x, y-plane: $\{(x, y) : x \in [1, 2], y \in [1, 2]\}$. ANSWER:



$\square$

Homework.

1. Read Section 1.1.
2. Write up the following exercises:
   Section 1.1: 8, 16, 22, 28, 33, 38.

New LaTeX commands

$\{\}$	`\{\}`
$\mathbb{N}$	`\bbn`
$\mathbb{Z}$	`\bbz`
$\mathbb{Q}$	`\bbq`
$\mathbb{R}$	`\bbr`
$\in$	`\in`
$\notin$	`\notin`
$\neq$	`\ne`
$\|A\|$	`|A|`
$\#A$	`\# A`
$\varnothing$	`\varnothing` (also `\emptyset`)