Sections 1.3 and 1.4: Subsets and power sets

Definition

The set A is a *subset* of the set B if every element of A is an element of B, and this is denoted by writing $A \subseteq B$.
Otherwise, if A contains an element that is not in B, then A is not a subset of B, and we write $A \nsubseteq B$.

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Examples:

1. $\{2,1\} \subseteq \{1,2\}$
2. $\{2,1\} \subseteq \{1,2,4\}$
3. $\{2,1\} \not\subseteq \{\{1,2\},4\}$
4. $\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$

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Note that $\varnothing \not\subseteq A$ only if $\varnothing$ contains an element that is not in A.
Since this is not the case, $\varnothing \subseteq A$. □

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2. If $A = \{\varnothing, \{a\}, \zeta\}$, then the subsets of A are

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so $\mathscr{P}(A) = \Big\{ \{\}, \{\varnothing\}, \{\{a\}\}, \{\zeta\}, \{\varnothing, \{a\}\}, \{\varnothing, \zeta\}, \{\{a\}, \zeta\}, \{\varnothing, \{a\}, \zeta\} \Big\}$.

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so $\mathscr{P}(A) = \big\{ \{\}, \{\varnothing\}, \{\{a\}\}, \{\zeta\}, \{\varnothing, \{a\}\}, \{\varnothing, \zeta\}, \{\{a\}, \zeta\}, \{\varnothing, \{a\}, \zeta\} \big\}$.
3. If $A = \varnothing$, then $\mathscr{P}(A) = \{\varnothing\}$ since the only subset of $\varnothing$ is $\varnothing$.

Theorem

If A is a finite set, then $|\mathscr{P}(A)| = 2^{|A|}$. In other words, if A is a finite set, then the number of subsets of A is $2^{|A|}$.

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Proof.

Subsets of A are constructed by selecting various elements of A.
In other words, for each element in A, there are two choices: include it in the subset, or exclude it.
The total number of choices is the product of the number of choices for each element, thus the total number of choices is

$$\underbrace{2 \cdot 2 \cdot 2 \cdots 2}_{|A|} = 2^{|A|}.$$

$\square$

Homework.

1. Read Sections 1.3 and 1.4.
2. Write up the following exercises.
   Section 1.3: 2, 3, 11, 12.
   Section 1.4: 5, 6, 8, 15, 17.

New LaTeX commands	
$\subseteq$	`\subseteq`
$\not\subseteq$	`\not\subseteq`
$\mathscr{P}(A)$	`\mathscr P(A)`