Section 1.8: Indexed sets

Definition

Suppose $A_1, A_2, \ldots, A_n$ are sets. Then

$$A_1 \cup A_2 \cup \cdots \cup A_n = \{x : x \in A_i \text{ for some } 1 \leq i \leq n\}.$$

This *finite union* is the union of finitely many sets.

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Example: $A_i = \{i\}$

$A_1 = \{1\}, A_2 = \{2\}, A_3 = \{3\}, \ldots, A_n = \{n\}$

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$\bigcup_{i=1}^{6} \{i\} = \{1\} \cup \{2\} \cup \{3\} \cup \cdots \cup \{6\} = \{1, 2, 3, 4, 5, 6\}.$

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A finite union may contain infinitely many elements.

Example: $A_i = [i-1, i]$

$A_1 = [0,1], A_2 = [1,2], A_3 = [2,3], A_4 = [3,4]$

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$$\bigcup_{i=1}^{n} [i-1, i] = [0,1] \cup [1,2] \cup \cdots \cup [n-1, n] = [0,n]$$

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Suppose $A_1, A_2, A_3, \dots$ is an infinite collection of sets. Then

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Warning: This does *not* include $i = \infty$.

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$$\bigcup_{i=1}^{\infty} \{i\} = \{1\} \cup \{2\} \cup \{3\} \cup \cdots = \{1, 2, 3, \ldots\} = \mathbb{N}.$$

Note: $\{\infty\}$ is *not* one of these sets (∞ is not a natural number).

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$A_1 = (-1, 1), A_2 = (-2, 2), A_3 = (-3, 3), A_4 = (-4, 4)$

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Note: $(-\infty, \infty)$ is *not* one of the sets in this union.

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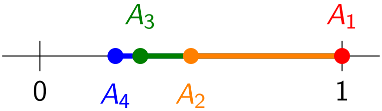

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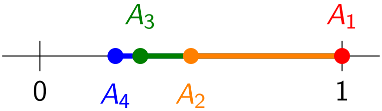

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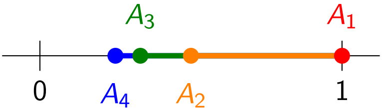

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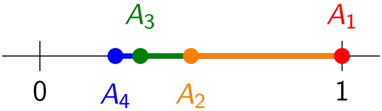

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Since $0 < \frac{1}{i}$ for every $i \in \mathbb{N}$, we know that $0 \notin A_i$ for any $i \in \mathbb{N}$.

Thus $0 \notin \bigcup_{i=1}^{\infty} \left[\frac{1}{i}, 1\right]$.

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Suppose $A_1, A_2, \ldots, A_n$ are sets. Then

$$A_1 \cap A_2 \cap \cdots \cap A_n = \{x : x \in A_i \text{ for every } 1 \leq i \leq n\}.$$

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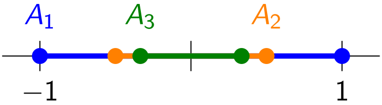
$$\bigcap_{i=1}^{\infty}(-i, i) = (-1, 1) \cap (-2, 2) \cap (-3, 3) \cap \cdots = (-1, 1).$$

Example: $A_i = \left[-\frac{1}{i}, \frac{1}{i}\right]$



$A_1 = [-1, 1]$, $A_2 = \left[-\frac{1}{2}, \frac{1}{2}\right]$, $A_3 = \left[-\frac{1}{3}, \frac{1}{3}\right]$, ...

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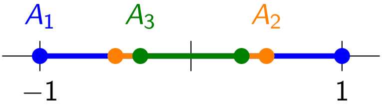

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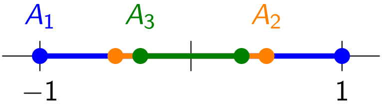

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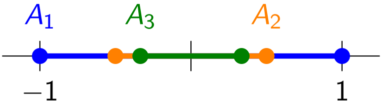

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Since $-\frac{1}{i} < 0 < \frac{1}{i}$ for every $i \in \mathbb{N}$, we know that $0 \in A_i$ for every $i \in \mathbb{N}$.

Thus $0 \in \bigcap_{i=1}^{\infty} \left[-\frac{1}{i}, \frac{1}{i}\right]$.

Homework.

1. Read Section 1.8.
2. Write up the following exercises.
   Section 1.8: 1, 3, 8, 9

New LaTeX commands

$\bigcup_{i=1}^n A_i$	`\bigcup_{i=1}^n A_i`
$\bigcap_{i=1}^\infty A_i$	`\bigcap_{i=1}^\infty A_i`
$\cdots \cup \dotsb$	`\cdots \cup \dotsb`

Dots for binary operations (e.g. $\cup$, $\cap$, $+$, $-$, etc.) should be centered. The most common commands to produce centered dots are `\cdots` (centered dots) or `\dotsb` (dots for binary operations).

When written in-line, the delimiters on these symbols appear to the right. To place the delimiters above/below, write these symbols in the `align` environment. (Do not place the `align` environment in math mode.)

```
\begin{align*}
\bigcup_{i=1}^n A_i
\end{align*}
```

$$\bigcup_{i=1}^n A_i$$

```
\begin{align*}
\bigcap_{i=1}^\infty A_i
\end{align*}
```

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