

Section 2.6

Logical statements II

P	Q	$P \Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	$P \Rightarrow Q$	$\sim Q$	$\sim P$
T	T	T	F	F
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

P	Q	$P \Rightarrow Q$	$\sim Q$	$\sim P$	$(\sim Q) \Rightarrow (\sim P)$
T	T	T	F	F	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

P	Q	$P \Rightarrow Q$	$\sim Q$	$\sim P$	$(\sim Q) \Rightarrow (\sim P)$
T	T	T	F	F	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

Definition

Two statements, X and Y , are *logically equivalent* if X and Y have identical columns in a truth table. That is, X is true if and only if Y is true.

The statements $P \Rightarrow Q$ and $(\sim Q) \Rightarrow (\sim P)$ are logically equivalent.

P	Q	$P \Rightarrow Q$	$\sim Q$	$\sim P$	$(\sim Q) \Rightarrow (\sim P)$
T	T	T	F	F	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

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Examples:

If it rains heavily, then the rivers will swell.

If the rivers did not swell, then it did not rain heavily.

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Examples:

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If the rivers did not swell, then it did not rain heavily.

$$(x \in A \Rightarrow x \in B) \Leftrightarrow (x \notin B \Rightarrow x \notin A).$$

$$(x \in A \Rightarrow x \notin \bar{A}) \Leftrightarrow (x \in \bar{A} \Rightarrow x \notin A).$$

Theorem (DeMorgan's Laws)

① $\sim (P \wedge Q) \Leftrightarrow (\sim P) \vee (\sim Q).$

② $\sim (P \vee Q) \Leftrightarrow (\sim P) \wedge (\sim Q).$

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Proof.

To show that the statements above are logically equivalent, we must show that these statements have identical columns in a truth table.

①

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F



Theorem (DeMorgan's Laws)

$$\textcircled{1} \sim (P \wedge Q) \Leftrightarrow (\sim P) \vee (\sim Q).$$

$$\textcircled{2} \sim (P \vee Q) \Leftrightarrow (\sim P) \wedge (\sim Q).$$

Proof.

To show that the statements above are logically equivalent, we must show that these statements have identical columns in a truth table.

$\textcircled{1}$

P	Q	$P \wedge Q$	$\sim (P \wedge Q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T



Theorem (DeMorgan's Laws)

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T	T	T	F	F	F
T	F	F	T	F	T
F	T	F	T	T	F
F	F	F	T	T	T



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P	Q	$P \wedge Q$	$\sim (P \wedge Q)$	$\sim P$	$\sim Q$	$(\sim P) \vee (\sim Q)$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T



Homework.

- 1 Read Section 2.6.
- 2 Write up the following exercises.
Section 2.6: 9, 11.

New L^AT_EX commands

Tables are created using the `tabular` environment.

The number of columns is specified immediately after the `\begin{tabular}` command. In this example, `{c|c|c}` creates a table with three centered columns. The vertical bar `|` places a vertical line between the columns. Columns can also be right (`r`) or left (`l`) justified. For example, the command `{rlllr}` would create a table with five columns, where the first and last columns are right justified, and the middle three are left justified.

Use `&` to separate text between columns. Use a double backslash to create a new line in the table. The `\hline` command creates a horizontal line.

```
\begin{tabular}{c|c|c}
P$ & $Q$ & $P \land Q$ \\
\hline
T & T & T \\
T & F & F \\
F & T & F \\
F & F & F
\end{tabular}
```

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F