Sections 2.7 and 2.8
Logical statements III

Definition

Let P and Q be statements. The statement "*there exists* P such that Q" is true if there is at least one case where P is true and Q is true. The statement is false if P is false whenever Q is true.

Definition

Let P and Q be statements. The statement "*there exists* P such that Q" is true if there is at least one case where P is true and Q is true. The statement is false if P is false whenever Q is true.

Examples:

1. There exists an integer n for which $2n$ is an integer.
   True (in fact, true for all integers).

Definition

Let P and Q be statements. The statement "*there exists* P such that Q" is true if there is at least one case where P is true and Q is true. The statement is false if P is false whenever Q is true.

Examples:

1. There exists an integer n for which $2n$ is an integer.
   True (in fact, true for all integers).
2. There exists an integer n for which $n/2$ is an integer.
   True (for some, though not all, integers).

Definition

Let P and Q be statements. The statement "*there exists* P such that Q" is true if there is at least one case where P is true and Q is true. The statement is false if P is false whenever Q is true.

Examples:

1. There exists an integer n for which $2n$ is an integer.
   True (in fact, true for all integers).
2. There exists an integer n for which $n/2$ is an integer.
   True (for some, though not all, integers).
3. There exists an integer n for which $n + \frac{1}{2}$ is an integer.
   False (for all integers).

Contrast with if-then statements.

1. There exists an integer n for which $2n$ is an integer. True.
   If n is an integer, then $2n$ is an integer. True.
2. There exists an integer n for which $n/2$ is an integer. True.
   If n is an integer, then $n/2$ is an integer. False.
3. There exists an integer n for which $n + \frac{1}{2}$ is an integer. False.
   If n is an integer, then $n + \frac{1}{2}$ is an integer. False.

Set theoretic interpretation.

1. There exists an integer n for which $2n$ is an integer. True.
   $\{n \in \mathbb{Z} : 2n \in \mathbb{Z}\} = \mathbb{Z}$
2. There exists an integer n for which $n/2$ is an integer. True.
   $\{n \in \mathbb{Z} : n/2 \in \mathbb{Z}\} = \{2m : m \in \mathbb{Z}\}$ (the set of even integers)
3. There exists an integer n for which $n + \frac{1}{2}$ is an integer. False.
   $\{n \in \mathbb{Z} : n + \frac{1}{2} \in \mathbb{Z}\} = \varnothing$

Set theoretic interpretation.

1. There exists an integer n for which $2n$ is an integer. True.
   $\{n \in \mathbb{Z} : 2n \in \mathbb{Z}\} = \mathbb{Z}$
2. There exists an integer n for which $n/2$ is an integer. True.
   $\{n \in \mathbb{Z} : n/2 \in \mathbb{Z}\} = \{2m : m \in \mathbb{Z}\}$ (the set of even integers)
3. There exists an integer n for which $n + \frac{1}{2}$ is an integer. False.
   $\{n \in \mathbb{Z} : n + \frac{1}{2} \in \mathbb{Z}\} = \varnothing$

A there-exists statement is true if and only if the "solution set" is non-empty.

Definition

Let P and Q be statements. The statement "Q *for all* P" (also "*for all* P, [we have] Q") is equivalent to the statement "if P, then Q".

Definition

Let P and Q be statements. The statement "Q *for all* P" (also "*for all* P, [we have] Q") is equivalent to the statement "if P, then Q".

Examples:

1. The number $2n$ is an integer for all $n \in \mathbb{Z}$.
   For all $n \in \mathbb{Z}$, the number $2n$ is an integer.
   If n is an integer, then $2n$ is an integer. True.

Definition

Let P and Q be statements. The statement "Q *for all* P" (also "*for all* P, [we have] Q") is equivalent to the statement "if P, then Q".

Examples:

1. The number $2n$ is an integer for all $n \in \mathbb{Z}$.
   For all $n \in \mathbb{Z}$, the number $2n$ is an integer.
   If n is an integer, then $2n$ is an integer. True.
2. The number $n/2$ is an integer for all $n \in \mathbb{Z}$.
   For all $n \in \mathbb{N}$, the number $n/2$ is an integer.
   If n is an integer, then $n/2$ is an integer. False.

Definition

Let P and Q be statements. The statement "Q *for all* P" (also "*for all* P, [we have] Q") is equivalent to the statement "if P, then Q".

Examples:

1. The number $2n$ is an integer for all $n \in \mathbb{Z}$.
   For all $n \in \mathbb{Z}$, the number $2n$ is an integer.
   If n is an integer, then $2n$ is an integer. True.
2. The number $n/2$ is an integer for all $n \in \mathbb{Z}$.
   For all $n \in \mathbb{N}$, the number $n/2$ is an integer.
   If n is an integer, then $n/2$ is an integer. False.
3. The number $n + \frac{1}{2}$ is an integer for all $n \in \mathbb{Z}$.
   For all $n \in \mathbb{N}$, the number $n + \frac{1}{2}$ is an integer.
   If n is an integer, then $n + \frac{1}{2}$ is an integer. False.

Existential quantifier	$\exists$	"there exists"
Universal quantifier	$\forall$	"for all" "for each" "for every" "for any"

The symbol $\forall$ is ambiguous. Refrain from using $\forall$ in formal writing. Write out the words instead.

Existential quantifier	$\exists$	"there exists"
Universal quantifier	$\forall$	"for all" "for each" "for every" "for any"

The symbol $\forall$ is ambiguous. Refrain from using $\forall$ in formal writing. Write out the words instead.

Everyone sat on the couch.

Each person sat on the couch.

Existential quantifier	$\exists$	"there exists"
Universal quantifier	$\forall$	"for all" "for each" "for every" "for any"

The symbol $\forall$ is ambiguous. Refrain from using $\forall$ in formal writing. Write out the words instead.

Everyone (together) sat on the couch (as one).

Each person (individually) sat on the couch (at some point).

For each $n \in \mathbb{Z}$, there exists an $D \in \mathbb{N}$ such that $D < n$.

For each $n \in \mathbb{Z}$, there exists an $D \in \mathbb{N}$ such that $D < n$.
If $n \in \mathbb{Z}$, then there exists an $D \in \mathbb{N}$ such that $D < n$.

For each $n \in \mathbb{Z}$, there exists an $D \in \mathbb{N}$ such that $D < n$.
If $n \in \mathbb{Z}$, then there exists an $D \in \mathbb{N}$ such that $D < n$.

This statement is false if "$n \in \mathbb{Z}$" is true, but "there exists an $D \in \mathbb{N}$ such that $D < n$" is false.

For each $n \in \mathbb{Z}$, there exists an $D \in \mathbb{N}$ such that $D < n$.
If $n \in \mathbb{Z}$, then there exists an $D \in \mathbb{N}$ such that $D < n$.

This statement is false if "$n \in \mathbb{Z}$" is true, but "there exists an $D \in \mathbb{N}$ such that $D < n$" is false.

Given $n \in \mathbb{Z}$, is it possible to find a $D \in \mathbb{N}$ that is less than n?

For each $n \in \mathbb{Z}$, there exists an $D \in \mathbb{N}$ such that $D < n$.
If $n \in \mathbb{Z}$, then there exists an $D \in \mathbb{N}$ such that $D < n$.

This statement is false if "$n \in \mathbb{Z}$" is true, but "there exists an $D \in \mathbb{N}$ such that $D < n$" is false.

Given $n \in \mathbb{Z}$, is it possible to find a $D \in \mathbb{N}$ that is less than n?

In general, no. If $n = -2$, then no such D exists.

(The original statement is false.)

For each $n \in \mathbb{Z}$, there exists a $D \in \mathbb{N}$ such that $D > n$.

For each $n \in \mathbb{Z}$, there exists a $D \in \mathbb{N}$ such that $D > n$.
If $n \in \mathbb{Z}$, then there exists a $D \in \mathbb{N}$ such that $D > n$.

For each $n \in \mathbb{Z}$, there exists a $D \in \mathbb{N}$ such that $D > n$.
If $n \in \mathbb{Z}$, then there exists a $D \in \mathbb{N}$ such that $D > n$.

This statement is false if "$n \in \mathbb{Z}$" is true, but "there exists a $D \in \mathbb{N}$ such that $D > n$" is false.

For each $n \in \mathbb{Z}$, there exists a $D \in \mathbb{N}$ such that $D > n$.
If $n \in \mathbb{Z}$, then there exists a $D \in \mathbb{N}$ such that $D > n$.

This statement is false if "$n \in \mathbb{Z}$" is true, but "there exists a $D \in \mathbb{N}$ such that $D > n$" is false.

Given $n \in \mathbb{Z}$, is it possible to find a $D \in \mathbb{N}$ that is greater than n?

For each $n \in \mathbb{Z}$, there exists a $D \in \mathbb{N}$ such that $D > n$.
If $n \in \mathbb{Z}$, then there exists a $D \in \mathbb{N}$ such that $D > n$.

This statement is false if "$n \in \mathbb{Z}$" is true, but "there exists a $D \in \mathbb{N}$ such that $D > n$" is false.

Given $n \in \mathbb{Z}$, is it possible to find a $D \in \mathbb{N}$ that is greater than n?

Yes. Given n, let $D = |n| + 1$.

(The original statement is true.)

For each $n \in \mathbb{Z}$, there exists a $D \in \mathbb{N}$ such that $D > n$.

True. Given n, let $D = |n| + 1$.

$$-3 < 4 \qquad 5 < 6 \qquad 10 < 11$$

For each $n \in \mathbb{Z}$, there exists a $D \in \mathbb{N}$ such that $D > n$.

True. Given n, let $D = |n| + 1$.

$$-3 < 4 \qquad 5 < 6 \qquad 10 < 11$$

The value D depends on n. There does not exist a number D that works for all n.

For each $n \in \mathbb{Z}$, there exists a $D \in \mathbb{N}$ such that $D > n$.

True. Given n, let $D = |n| + 1$.

$$-3 < 4 \qquad 5 < 6 \qquad 10 < 11$$

The value D depends on n. There does not exist a number D that works for all n.

There exists a $D \in \mathbb{N}$ such that $D > n$ for all $n \in \mathbb{Z}$.

Given $D \in \mathbb{N}$, is $D > n$ for all $n \in \mathbb{Z}$?

For each $n \in \mathbb{Z}$, there exists a $D \in \mathbb{N}$ such that $D > n$.

True. Given n, let $D = |n| + 1$.

$$-3 < 4 \qquad 5 < 6 \qquad 10 < 11$$

The value D depends on n. There does not exist a number D that works for all n.

There exists a $D \in \mathbb{N}$ such that $D > n$ for all $n \in \mathbb{Z}$. (False.)

Given $D \in \mathbb{N}$, is $D > n$ for all $n \in \mathbb{Z}$?

No. Given D, we have $D \not> D + 1$ and $D + 1 \in \mathbb{Z}$.

For each $n \in \mathbb{Z}$, there exists a $D \in \mathbb{N}$ such that $D > n$. (True.)
The value D depends on n.

There exists a $D \in \mathbb{N}$ such that $\sin(n) < D$ for all $n \in \mathbb{Z}$.

For each $n \in \mathbb{Z}$, there exists a $D \in \mathbb{N}$ such that $D > n$. (True.)
The value D depends on n.

There exists a $D \in \mathbb{N}$ such that $\sin(n) < D$ for all $n \in \mathbb{Z}$.

True,

$$\sin(n) < 2$$

so $D = 2$ satisfies this statement.

In this case, D is independent of n. The same number D works for every n.

Homework.

1. Read Sections 2.7 and 2.8.
2. Write up the following exercises:
   Section 2.7: 1–6. If a statement is false, explain why (give an example, if possible).

New LaTeX commands

$\forall$	`\forall`
$\exists$	`\exists`