# MATH 2001

**Definition 1.** Let $a$ and $b$ be integers. We say $a$ *divides* $b$ if there exists an integer $c$ for which $b = ac$, and we write $a \mid b$.

**Definition 2.** An integer $a$ is *even* if there exists an integer $n$ for which $a = 2n$.

**Definition 3.** An integer $a$ is *odd* if there exists an integer $n$ for which $a = 2n + 1$.

**Exercises:**

(1) Prove that the sum of two even integers is even.
(2) Prove that the sum of two odd integers is even.
(3) Prove that the product of two odd integers is odd.
(4) Prove that the product of an even integer and an odd integer is even.
(5) Prove that if $a \mid b$, then $a \mid b^2$.
(6) Prove that if $a \mid b$ and $b \mid c$, then $a \mid c$.
(7) Prove that if $a$ and $b$ are positive integers for which $a \mid b$ and $b \mid a$, then $a = b$.
(8) Prove that if $a = 3b + 1$ for some integer $b$, then $3 \mid (a^2 + a + 1)$.
(9) Prove that if $a$ is an odd integer for which $a = 3b + 1$ or $a = 3b + 2$ for some integer $b$, then $24 \mid (a^2 - 1)$.