# MATH 2001

**Definition.** Let $A$ be a set. A *basis for a topology* on $A$ is a collection $\mathscr{B}$ of subsets of $A$ (called *basis elements*) that satisfies the following.

(1) For each $x \in A$, there exists a $B \in \mathscr{B}$ such that $x \in B$.
(2) If $B_1$ and $B_2$ are any two elements in $\mathscr{B}$ and $x \in B_1 \cap B_2$, then there exists a $B_3 \in \mathscr{B}$ such that $x \in B_3$ and $B_3 \subseteq B_1 \cap B_2$.

$\star$ **Exercise.** In each of the following cases, prove that $\mathscr{B}$ is a basis for a topology on $A$, or explain why it is not.

a) $A = \{a, b, c, d, e\}$, and $\mathscr{B} = \{\{a, b\}, \{c, d\}, \{c, d, e\}\}$.

b) $A$ is a set, and $\mathscr{B} = \mathscr{P}(A)$.

c) $A$ is a set, and $\mathscr{B} = \{X \in \mathscr{P}(A) : |X| = 3\}$.

d) $A = \mathbb{R}$, and $\mathscr{B} = \{(a, b) \subset \mathbb{R} : a, b \in \mathbb{R}\}$. (The interval $(a, b)$ is the set $\{x \in \mathbb{R} : a < x < b\}$.)

e) $A = \mathbb{R}$, and $\mathscr{B} = \{[a, b) \subset \mathbb{R} : a, b \in \mathbb{R}\}$. (The interval $[a, b)$ is the set $\{x \in \mathbb{R} : a \leq x < b\}$.)

f) $A = \mathbb{R}$, and $\mathscr{B} = \{[a, b] : a, b \in \mathbb{R}\}$. (The interval $[a, b]$ is the set $\{x \in \mathbb{R} : a \leq x \leq b\}$.)

g) $A = \mathbb{R}$, and $\mathscr{B} = \{(a - 1, a + 1) \subset \mathbb{R} : a \in \mathbb{R}\}$.

h) $A = \mathbb{R}$, and $\mathscr{B} = \left\{\overline{\{x\}} : x \in \mathbb{R}\right\}$.

i) $A = \mathbb{R}$, and $\mathscr{B} = \left\{\overline{C} : C \subset \mathbb{R}, C \text{ is a finite set}\right\}$. (That is, $B \in \mathscr{B}$ if $B = \mathbb{R} - C$ for some finite set $C$.)