MATH 2001

So far this semester, we have proved a number of statements using *direct proofs*. The direct proof is a common method for proving "if P, then Q" statements, and the general outline for the proof is as follows.

General outline of a direct proof of $P \Rightarrow Q$:

Proof. Suppose P.
... (apply theorems or definitions, or perform algebraic manipulations) ...
Therefore Q. □

We call this proof method "direct" because we move in a linear fashion from the left side of the statement to the right.

Example 1. Let a, b, and c be integers. Prove that if $a \mid b$ and $a \mid c$, then $a \mid (b - c)$.

Definition (Divides)**.**

Proof. □

Example 2. Prove that if $n \in \mathbb{Z}$, then $n^2 + 3n + 4$ is even.

Definition (Even/odd)**.**

Proof. □

Our double containment arguments have also been direct proofs.

General outline of a double containment proof:

Proof. To prove $A = B$, we show that $A \subseteq B$ and $B \subseteq A$.
($A \subseteq B$) Suppose $x \in A$. ... Therefore $x \in B$.
($B \subseteq A$) Suppose $x \in B$. ... Therefore $x \in A$.
Thus $A = B$. □

Example 3. Prove that the set of odd integers is equal to the set $\{a^2 - b^2 : a + b \text{ is odd}\}$.

Proof. □

Recall that the statement $P \Rightarrow Q$ is equivalent to its contrapositive $(\sim Q) \Rightarrow (\sim P)$.

General outline of a proof of $P \Rightarrow Q$ via contrapositive:

Proof. Suppose $(\sim Q)$. ... Therefore $(\sim P)$. Thus $P \Rightarrow Q$. □

Example 4. Prove that $\displaystyle\bigcap_{x \in \mathbb{R}, x > 0} (-2x, x^{-1}] = \{0\}$.

Proof. □

Example 5. Prove that if $x^3 + yx^2 \leq x^3 + xy^2$, then $y \leq x$.

Proof. □

Homework. Due Tuesday, October 6.

Chapter 4: 11, 14. Chapter 5: 4, 7.

Suggested reading: Sections 4.3, 4.4, and 5.1.