MATH 2001

Recall that the statement $P \Rightarrow Q$ is equivalent to its contrapositive $(\sim Q) \Rightarrow (\sim P)$.

General outline of a proof of $P \Rightarrow Q$ via contrapositive:

Proof. Suppose $(\sim Q)$. ... Therefore $(\sim P)$. Thus $P \Rightarrow Q$. □

Example 1. Prove that if $y^3 + yx^2 \leq x^3 + xy^2$, then $y \leq x$.

Proof.

□

Example 2. Prove, by contrapositive, that if $x^3 - 1$ is even, then x is odd.

Proof.

□

A third method of proof is the *proof by contradiction*. Suppose we wanted to prove the statement P. To prove the statement by contradiction, we show that if P is false, then absurdity ensues.

General outline of a proof of P by contradiction:

Proof. Suppose $(\sim P)$. ... Contradiction! Therefore P. $\square$

Example 3. Prove that there is no integer that is both even and odd.

Proof.

$\square$

Example 4. Prove that $\sqrt{2}$ is irrational.

Proof.

$\square$

Homework. Due 6pm on Friday, October 9.
Chapter 6: 3, 8, 14.
Suggested reading: Chapter 6.