MATH 2001
BINOMIAL COEFFICIENTS

Example 1. Prove that $\binom{n}{n-k} = \binom{n}{k}$.

Proof.

□

Example 2. Prove that the sum of values on the n-th row of Pascal's triangle is 2^n.

Proof.

□

Example 3. Prove that the sum of values on the n-th shallow diagonal is the n-th Fibonacci number.

Proof.

□

Example 4. Prove that the alternating sum of values on the n-th row of Pascal's triangle is 0.

Proof.

□

Homework. Four proofs due 6pm on Tuesday, October 27.

(1) Prove that if $a, b, c \in \mathbb{N}$ and $c \leq b \leq a$, then $\binom{a}{b}\binom{b}{c} = \binom{a}{b-c}\binom{a-b+c}{c}$.
(2) Prove that $\sum_{j=0}^{n}\sum_{k=0}^{j}\binom{j}{k} = 2^{n+1} - 1$. In other words, prove that the sum of all values in the first n rows (row 0 up to row n) is equal to $2^{n+1} - 1$.
(3) Prove that $\sum_{k=0}^{n}\binom{n}{k}^2 = \binom{2n}{n}$.

Suggested reading: Chapter 3.