MATH 2001

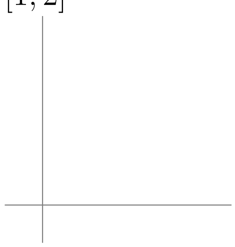
(1) Let $A = \{a, b, \{a, b\}, \mathbb{Z}\}$. Determine whether the following statements are true or false.

a.	**T**	**F**	:	$\varnothing \in A$	g.	**T**	**F**	:	$\varnothing \subseteq A$
b.	**T**	**F**	:	$\varnothing \in \mathscr{P}(A)$	h.	**T**	**F**	:	$\varnothing \subseteq \mathscr{P}(A)$
c.	**T**	**F**	:	$\{a, b\} \in A$	i.	**T**	**F**	:	$\{a, b\} \subseteq A$
d.	**T**	**F**	:	$\{a, b\} \in \mathscr{P}(A)$	j.	**T**	**F**	:	$\{a, b\} \subseteq \mathscr{P}(A)$
e.	**T**	**F**	:	$\{a, b\} \in A^2$	k.	**T**	**F**	:	$\{a, b\} \subseteq A^2$
f.	**T**	**F**	:	$1 \in A$	l.	**T**	**F**	:	$1 \subseteq A$

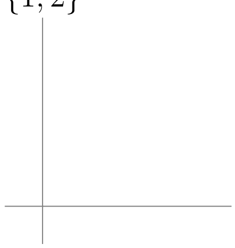
For each false statement, give a brief explanation as to why the statement is not true.

(2) Sketch each of the following sets in the x, y-plane.

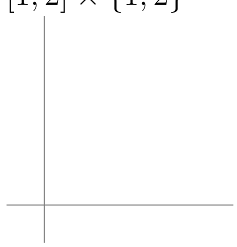
a. $[1, 2]^2$



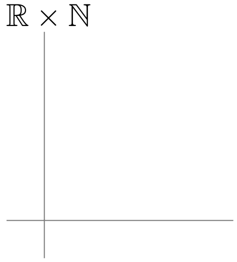
b. $\{1, 2\}^2$



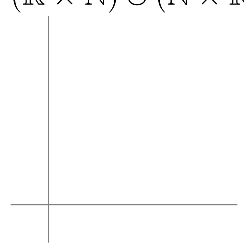
c. $[1, 2] \times \{1, 2\}$



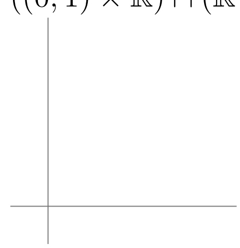
d. $\mathbb{R} \times \mathbb{N}$



e. $(\mathbb{R} \times \mathbb{N}) \cup (\mathbb{N} \times \mathbb{R})$



f. $\overline{((0, 1) \times \mathbb{R})} \cap (\mathbb{R} \times (1, 2))$



Let us recall a few definitions.

Definition. Two sets, A and B, are *equal* if they contain exactly the same elements.

Definition. The set A is a *subset* of the set B if every element of A is also an element of B.

These statements (and all definitions) serve two purposes. On one hand, definitions allow for a single word to encapsulate a more complicated idea. For example, if you are given that $A = B$, then you *know* that A and B contain exactly the same elements.

On the other hand, if you want to show that certain objects are a keyword, the definition gives conditions which the objects must satisfy. Going back to the equality example, if you want to *show* that $A = B$, then you must first show that A and B contain exactly the same elements. In other words, given two sets A and B, if you show that A and B contain exactly the same elements, then you can conclude that $A = B$.

The definitions above translate statements about sets ($A = B$ or $A \subseteq B$) into statements about elements. One of the goals of this exercise is to make the statement about elements more precise.

(1) Suppose $A = B$. If $x \in A$, what can you conclude?

(2) Write a similar statement for the case where $A = B$ and $x \in B$.

(3) Using these two new statements, what condition(s) would you need to satisfy in order to show that $A = B$? In other words, rewrite the "show" statement I have above using precise statements regarding the elements in A and B.

(4) Suppose $A \subseteq B$. What do you know about the elements of A and B. Specifically, what can you say about $x \in A$?

(5) What do you need to show in order to conclude that $A \subseteq B$?

(6) Suppose you know that $A = B$. Use the statements you have written above to show that $A \subseteq B$ and $B \subseteq A$.

(7) Suppose you know that $A \subseteq B$ and $B \subseteq A$. Use the statements you have written above to show that $A = B$.

What you have shown is that the statement "$A = B$" is equivalent to the statement "$A \subseteq B$ and $B \subseteq A$". When trying to show that two sets are equal, it is often easier to use this *double containment* statement to break the work up into two steps: to show that $A = B$, first show that $A \subseteq B$, then show that $B \subseteq A$.