Definition

Set and element

Sets

Definition

Empty set (null set)

Sets

Definition

Ordered pair / n-tuple

Sets

Definition

Subset

Sets

Definition

Power set

Sets

Definition

Cardinality of a set

Sets

Definition

Finite and infinite sets

Sets

Definition

Cartesian product

Sets

Definition

Proper subset

Sets

Definition

Partition

Sets

The *cardinality* of a set is the number of elements in the set.
The cardinality of the set A is denoted by $|A|$ or $\#A$.

A *set* is a collection of objects; the objects in the set are called *elements*.
If A is a set, and a is an element of A, then we write $a \in A$.

A set is *finite* if its cardinality is finite, that is, the set contains finitely many elements.
A set is *infinite* if it contains infinitely many elements.

The *empty set* is the set that contains zero elements.
The empty set is denoted by $\varnothing$.

If A and B are sets, the *cartesian product* $A \times B$ is the set of ordered pairs where the first element is from A, and the second is from B.

$$A \times B = \{(a, b) : a \in A, b \in B\}.$$

$$(a, b) \in A \times B \quad \Leftrightarrow \quad a \in A \text{ and } b \in B$$

An *ordered pair* is an ordered list of two elements.
More generally, an *ordered n-tuple* is an ordered list of n elements. The standard notation is to use a comma separated list enclosed by parenthesis: $(a_1, a_2, \ldots, a_n)$.

The set B is a *proper* subset of A if B is a subset of A that is not equal A, and we write $B \subset A$.

$$B \subset A \quad \Leftrightarrow \quad B \subseteq A \text{ and } B \neq A$$

A set A is a *subset* of a set B if every element of A is an element of B.

$$A \subseteq B \quad \Leftrightarrow \quad x \in A \Rightarrow x \in B$$

Let A be a set, and let $P \subseteq \mathscr{P}(A)$. The set P is a *partition of A* if

1. $\bigcup_{X \in P} X = A$;
2. if $X_1, X_2 \in P$, then $X_1 \cap X_2 = \varnothing \Leftrightarrow X_1 \neq X_2$.

The *power set* of a set A is the set of all subsets of A.
The power set of A is denoted by $\mathscr{P}(A)$.

$$\mathscr{P}(A) = \{B : B \subseteq A\}$$

Definition

Set equality

Sets

Definition

Finite and infinite union

Sets

Definition

Finite and infinite intersection

Sets

Definition

Disjoint sets

Sets

Definition

Complement

Sets

Definition

Union

Sets

Definition

Intersection

Sets

Definition

Set difference

Sets

Theorem

Double containment principle

Sets

Theorem

De Morgan's laws

Sets

The *union* of two sets, A and B, is the set of all element in A or in B.
The union of these sets is denoted by $A \cup B$.

$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$

$x \in A \cup B \quad \Leftrightarrow \quad x \in A$ or $x \in B$

Two sets, A and B, are *equal* if all the elements of A are elements of B and vice versa.

$A = B \quad \Leftrightarrow \quad x \in A$ if and only if $x \in B$

The *intersection* of two sets, A and B, is the set of all element in A and in B.
The intersection of these sets is denoted by $A \cap B$.

$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$

$x \in A \cap B \quad \Leftrightarrow \quad x \in A$ and $x \in B$

A *finite union* is the union of finitely many sets. An *infinite union* is the union of infinitely many sets.
Let $A_1, A_2, A_3, \ldots$ be sets, then

$$\bigcup_{i=1}^{n} A_i = \{x : x \in A_i \text{ for some } 1 \leq i \leq n\}$$
$$\bigcup_{i \in \mathbb{N}} A_i = \{x : x \in A_i \text{ for some } i \in \mathbb{N}\}$$

If A and B are sets, the, is the difference $A - B$ is the set of elements in A that are not in B.

$$A - B = \{x : x \in A \text{ and } x \notin B\}$$

$x \in A - B \quad \Leftrightarrow \quad x \in A$ and $x \notin B$

A *finite intersection* is the intersection of finitely many sets. An *infinite intersection* is the intersection of infinitely many sets.
Let $A_1, A_2, A_3, \ldots$ be sets, then

$$\bigcap_{i=1}^{n} A_i = \{x : x \in A_i \text{ for all } 1 \leq i \leq n\}$$
$$\bigcap_{i \in \mathbb{N}} A_i = \{x : x \in A_i \text{ for all } i \in \mathbb{N}\}$$

Let A and B be sets. Then $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.

$A = B \quad \Leftrightarrow \quad A \subseteq B$ and $B \subseteq A$

Two sets, A and B, are *disjoint* if $A \cap B = \varnothing$.

For any sets A and B,

$$(A \cap B)^c = A^c \cup B^c,$$
$$(A \cup B)^c = A^c \cap B^c.$$

The *complement* of a set A is the set of all elements that are not in A, and is denoted by A^c or $\overline{A}$.
If $A \subseteq B$, then the *complement of A in B* is the set of elements in B that are not in A, i.e. $A^c = B - A$.

$$A^c = \{x : x \notin A\}$$

$x \in A^c \quad \Leftrightarrow \quad x \notin A$

Definition

Statement

Logic

Definition

Logical statement

Logic

Definition

Logical and

Logic

Definition

Logical or

Logic

Definition

Logical negation

Logic

Definition

Implies

Logic

Definition

Converse

Logic

Definition

If and only if

Logic

Definition

Contrapositive

Logic

Definition

Universal quantifier: for all

Logic

A *statement* is a sentence or mathematical expression that is definitely true or definitely false.

A *statement* is a sentence or mathematical expression that is definitively true or definitively false.

The statement “P *or* Q” is true if P is true or Q is true (or both statements are true). The statement “P and Q” is false only if both P is false and Q is false.

$P \vee Q$ is true $\Leftrightarrow$ P is true or Q is true

The statement “P *and* Q” is true if both P is true and Q is true. Otherwise “P and Q” is false.

$P \wedge Q$ is true $\Leftrightarrow$ P is true and Q is true

The statement “P *implies* Q” ($P \Rightarrow Q$) is false if P is true and Q is false. Otherwise the statement is true.

$P \Rightarrow Q$ is false $\Leftrightarrow$ P is false and Q is true

The *negation* of a statement P is the statement $\neg P$. The statement $\neg P$ is true if P is true. The statement of $\neg P$ is false if P is true.

P is true (resp. false) $\Leftrightarrow$ $\neg P$ is false (resp. true)

The statement “P *if and only if* Q” ($P \Leftrightarrow Q$) is equivalent to the statement $(P \Rightarrow Q) \wedge (Q \Rightarrow P)$. In other words, $P \Leftrightarrow Q$ is true if both $P \Rightarrow Q$ and $Q \Rightarrow P$ are true.

$P \Leftrightarrow Q$ $\Leftrightarrow$ $(P \Rightarrow Q) \wedge (Q \Rightarrow P)$

The *converse* of $P \Rightarrow Q$ is the statement $Q \Rightarrow P$. In general, these two statements are independent, meaning that the truthfulness of one statement does not determine the truthfulness of the other.

The *for all/each/every/any* statement takes the form: “for all P, we have Q.” In other words, Q is true whenever P is true. In this light, “for all” statements can often be reworded as “if-then” statements (and vice versa).

$\forall P$, we have Q $\Leftrightarrow$ $P \Rightarrow Q$

The *contrapositive* of the statement “if P, then Q” is the statement “if $\neg Q$, then $\neg P$”. These statements are equivalent, meaning that they are either both true or both false.

$P \Rightarrow Q$ $\Leftrightarrow$ $\neg Q \Rightarrow \neg P$

Definition

Existential quantifier: there exists

Logic

Negation of $P \wedge Q$

Logic

Negation of $P \vee Q$

Logic

Negation of $P \Rightarrow Q$

Logic

Negation of $\forall P,$ we have Q

Logic

Negation of $\exists P$ such that Q

Logic

Definition

Theorem

Logic

Definition

Proof

Logic

Definition

Definition

Logic

Definition

List and entries

Counting

The *there exists* statement takes the form: "there exists P such that Q." This statement is true if there is at least one case where P is true and Q is true. (It maybe that there are many cases where P is false but Q is true.)

$\exists P$ such that Q $\Leftrightarrow$ it is sometimes the case that $P \Rightarrow Q$

$$\neg(P \wedge Q) = \neg P \vee \neg Q$$

$$\neg(P \vee Q) = \neg P \wedge \neg Q$$

$$\neg(P \Rightarrow Q) = P \wedge \neg Q$$

$$\neg(\forall P, \text{ we have } Q) = \exists P, \text{ such that } \neg Q$$

$$\neg(\exists P, \text{ such that } Q) = \forall P \text{ we have } \neg Q$$

A *theorem* is a mathematical statement that is true and can be (and has been) verified as true.

A *proof* of a theorem is a written verification that shows that the theorem is definitely and unequivocally true.

A *definition* is an exact, unambiguous explanation of the meaning of a mathematical word or phrase.

A *list* is an ordered sequence of objects. The objects in the list are called *entries*. Unlike sets, the order of entries matters, and entries may be repeated.

Definition

List length

Counting

Definition

Empty list

Counting

Definition

Factorial

Counting

Theorem

Binomial theorem

Counting

Theorem

Addition principle

Counting

Definition

List equality

Counting

Theorem

Multiplication principle

Counting

Definition

n choose k

Counting

Theorem

Inclusion-exclusion

Counting

Definition

Even

Counting

Two lists L and M are *equal* if they have the same length, and the i-th entry of L is the i-th entry of M.

The *length* of a list is the number of entries in the list.

Suppose in making a list of length n there are a_i possible choices for the i-th entry. Then the total number of different lists that can be made in this way is $a_1 a_2 a_3 \cdots a_n$.

The *empty list* is the list with no entries, and is denoted by $()$.

If n and k are integers, and $0 \leq k \leq n$, then

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}.$$

If n is a non-negative integer, then $n!$ is the number of non-repetitive lists of length n that can be made from n symbols. Thus $0! = 1$, and if $n > 1$, then $n!$ is the product of all integers from 1 to n. That is, if $n > 1$, then $n! = n(n-1)(n-2)\cdots 2 \cdot 1$.

If A and B are sets, then $|A \cup B| = |A| + |B| - |A \cap B|$.

If n is a non-negative integer, then

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}.$$

An integer a is *even* if there exists an integer b such that $a = 2b$.

---

a is even $\Leftrightarrow$ $a = 2b$ for some $b \in \mathbb{Z}$

If $A_1, A_2, \ldots, A_n$ are disjoint sets, then $|A_1 \cup A_2 \cup \cdots \cup A_n| = |A_1| + |A_2| + \cdots + |A_n|$.

Definition

Odd

Counting

Definition

Divides

Counting

Definition

Parity

Counting

Definition

Prime

Counting

Definition

Composite

Counting

Definition

Greatest common divisor

Counting

Definition

Least common multiple

Counting

Theorem

Well-ordering principle

Counting

Theorem

Division algorithm

Counting

Definition

Basis for a topology

Topology

If a and b are integers, then b *divides* a if there exists an integer q such that $a = qb$. In this case, b is a *divisor* of a, and a is a *multiple* of b.

$b \mid a$	$\Leftrightarrow$	$a = qb$ for some $q \in \mathbb{Z}$

An integer a is *odd* if there exists an integer b such that $a = 2b + 1$.

a is odd	$\Leftrightarrow$	$a = 2b + 1$ for some $b \in \mathbb{Z}$

A positive integer $p > 1$ is *prime* if the only divisors of p are 1 and p.

$p > 1$ is *prime*	$\Leftrightarrow$	a has exactly two positive divisors: 1 and p

Two integers have the *same parity* if they are both even or both odd. Otherwise they have *opposite parity*.

The *greatest common divisor* of two integers a and b, denoted $\gcd(a, b)$, is the largest integer that divides both a and b.

A positive integer a is *composite* if there exists a positive integer $b > 1$ satisfying $b \mid a$.

$a > 1$ is composite	$\Leftrightarrow$	$b \mid a$ and $1 < b < a$

Every non-empty subset of $\mathbb{N}$ contains a least element.

The *least common multiple* of two integers a and b, denoted $\operatorname{lcm}(a, b)$, is the smallest positive integer is a multiple of both a and b.

Let A be a set, and let $\mathcal{B} \subseteq \mathscr{P}(A)$. The set $\mathcal{B}$ is a *basis for a topology on* A if the following are satisfied.

1. If $x \in A$, then there exists $B \in \mathcal{B}$ such that $x \in B$.
2. If $B_1, B_2 \in \mathcal{B}$ and $x \in B_1 \cap B_2$, then there exists $B_3 \in \mathcal{B}$ such that $x \in B_3$ and $B_3 \subseteq B_1 \cap B_2$.

Given integers a and b with $b > 0$, there exist unique integers q and r that satisfy $a = bq + r$, where $0 \leq r < b$.

Definition

Open set

Topology

Definition

Closed set

Topology

Definition

Relation on a set

Relations

Definition

Reflexive

Relations

Definition

Symmetric

Relations

Definition

Transitive

Relations

Definition

Equivalence relation

Relations

Definition

Equivalence class

Relations

Definition

Relation between sets

Relations

Definition

Inverse relation

Relations

Let A be a set, let $\mathcal{B}$ be a basis for a topology on A, and let $U \subseteq A$. The set U is *closed* if U^c is open.

Let A be a set, let $\mathcal{B}$ be a basis for a topology on A, and let $U \subseteq A$. The set U is *open* if for each $x \in U$, there exists $B \in \mathcal{B}$ such that $x \in B$ and $B \subseteq U$.

Let R be a relation on A. The relation R is *reflexive* if $a \in A$ implies that $(a, a) \in R$.

Let A be a set. The set R is a *relation on A* if $R \subseteq A^2$.

Let R be a relation on A. The relation R is *transitive* if $(a, b), (b, c) \in R$ implies that $(a, c) \in R$.

Let R be a relation on A. The relation R is *symmetric* if $(a, b) \in R$ implies that $(b, a) \in R$.

Let R be an equivalence relation on A, and let $a \in A$. The *equivalence class of a* is the set

$$[a] = \{b \in A : (a, b) \in R\}.$$

$x \in [a] \quad \Leftrightarrow \quad (a, x) \in R$

Let R be a relation on A. The relation R is an *equivalence relation* (on A) if it is reflexive, symmetric, and transitive.

Let R be a relation from A to B. The *inverse* of R is the relation from B to A given by

$$R^{-1} = \{(b, a) : (a, b) \in R\}.$$

$(x, y) \in R^{-1} \quad \Leftrightarrow \quad (y, x) \in R$

Let A and B be sets. The set R is a *relation from A to B* if $R \subseteq A \times B$.

Definition

Function

Relations

Definition

Domain/codomain/image

Relations

Definition

Image of a set

Relations

Definition

Inverse image of a set

Relations

Let f be a function from A to B. The *domain* of f is A. The *codomain* of f is B, and the *image* of f is the set $\{b \in B : (a, b) \in f\}$. In other words, the image of f is the set $\{f(a) : a \in A\}$.

Let R be a relation from A to B. The relation R is a *function* if for each $a \in A$, R contains a unique element of the form (a, b). In this case, we write $R(a) = b$.

Let f be a function from A to B, and let $V \subseteq B$. Then the *inverse image of* V (or *preimage of* V) is the set

$$f^{-1}(V) = \{x \in A : f(x) \in V\}.$$

$x \in f^{-1}(V)$	$\Rightarrow$	$f(x) \in V$
$y \in V$	$\Rightarrow$	$y = f(x)$ for some $x \in f^{-1}(V)$

Let f be a function from A to B, and let $U \subseteq A$. Then the *image of* U is the set

$$f(U) = \{f(x) \in B : x \in U\}.$$

$y \in f(U)$	$\Rightarrow$	$y = f(x)$ for some $x \in U$
$x \in U$	$\Rightarrow$	$f(x) \in f(U)$