# MATH 2001
# QUIZ 10

(1) (1 pt) Write your name in the top right corner of the page.

(2) (6 pts) Recall that *a basis of a topology on* $A$ is defined as follows.

**Definition.** Let $A$ be a set, and let $\mathcal{B}$ be a set of subsets of $A$. The set $\mathcal{B}$ is a *basis for a topology on* $A$ if the following properties are satisfied.

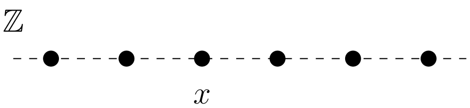
i For each $x \in A$, there exists a $B \in \mathcal{B}$ such that $x \in B$.

ii If $B_1, B_2 \in \mathcal{B}$, then for each $x \in B_1 \cap B_2$, there exists a $B_3 \in \mathcal{B}$ such that $x \in B_3$ and $B_3 \subseteq B_1 \cap B_2$.

Complete the following exercises to determine if $\mathcal{B} = \{\{n, n+1, n+2\} : n \in \mathbb{Z}\}$ is a basis for a topology on $A = \mathbb{Z}$.

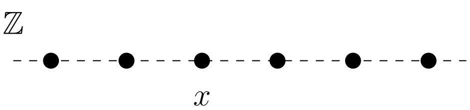
(a) Let $\mathcal{B} = \{\{n, n+1, n+2\} : n \in \mathbb{Z}\}$, and suppose $x \in \mathbb{Z}$.

Does there exist a $B \in \mathcal{B}$ such that $x \in B$? **If yes**, give a $B$ that contains $x$ **and** draw $B$ in the picture below. **If not**, give a counterexample, or briefly explain why no such $B$ exists.




Yes. Not drawn, but $\{x-2, x-1, x\}$, $\{x-1, x, x+1\}$, and $\{x, x+1, x+2\}$ are all valid basis elements.

(b) Suppose $B_1, B_2 \in \mathcal{B}$ and that $B_1 \cap B_2 \neq \varnothing$. For each $x \in B_1 \cap B_2$, does there exist a $B_3 \in \mathcal{B}$ such that $x \in B_3$ and $B_3 \subseteq B_1 \cap B_2$? **If yes**, give a brief explanation. **If not**, draw a $B_1$, $B_2$, and $B_3$ in the picture below that do not satisfy the condition.




No. For example if $B_1 = \{x-1, x, x+1\}$ and $B_2 = \{x, x+1, x+2\}$, then there does not exist a $B_3$ for which $x \in B_3$ and $B_3 \subseteq B_1 \cap B_2$. More generally, any $B_1$ and $B_2$ where $|B_1 \cap B_2| < 3$ yields a counterexample.

(3) (3 pts) Let $A$ be a set, let $\mathcal{B}$ be a basis for a topology on $A$, and let $U$ be a subset of $A$. State what it means for $U$ to be an *open set*.

The set $U$ is open if for each $x \in U$, there exists $B \in \mathcal{B}$ such that $x \in B$ and $B \subseteq U$.

*Date*: May 15, 2016.