MATH 2001
QUIZ 12

(1) (1 pt) Write your name in the top right corner of the page.

(2) (2 pts each) Complete each of the following definitions.

(a) A function $f\colon A \to B$ is *injective* if …

$f(a) = f(b)$ implies $a = b$.

(b) A function $f\colon A \to B$ is *surjective* if …

for each $b \in B$, there exists $a \in A$ such that $f(a) = b$.

(c) Let $f\colon A \to B$ be a function, and let $U \subseteq A$. Then

$$f(U) = \{f(x) \in B\colon x \in U\}$$

(d) Let $f\colon A \to B$ be a function, and let $V \subseteq B$. Then

$$f^{-1}(V) = \{x \in A\colon f(x) \in V\}$$

(3) (6 pts) Let $A = \{-1, 0, 1\}$, and consider the function $f\colon A^2 \to A$ defined by $f(x, y) = xy$.

(a) The function f **is** / **is not** (circle one) injective.

Is not.

(b) Justify your claim in part (a) by giving explicit values of the function. You don't have to write out the value of the function at every point—just give enough to confirm that the function is or isn't injective. (No need to write in full sentences either.)

$f(0, 0) = 0$ and $f(1, 0) = 0$ (but $(0, 0) \neq (1, 0)$).

(c) The function f **is** / **is not** (circle one) surjective.

Is.

(d) Justify your claim in part (c) by giving explicit values of the function.

$f(-1, 1) = -1$, $f(0, 0) = 0$, and $f(1, 1) = 1$.

(continued on reverse)

Date: May 4, 2016.

(4) (2 pts each) Let $A = \{-1, 0, 1\}$, and consider the function $f\colon A^2 \to \mathbb{Q}$ defined by $f(x, y) = x + y$. Write out each of the sets explicitly.

(a) $f^{-1}(\mathbb{N}) =$

$$\{(0, 1), (1, 0), (1, 1)\}$$

(b) $f(A^2) =$

$$\{-2, -1, 0, 1, 2\}$$