

MATH 2001
QUIZ 2

1. (1 pt) Write your name in the top *right* corner of the page.
2. (6 pts) Answer each question with the precise definition of each highlighted term. Write your answer as a **complete sentence or sentences**. You may use set-builder notation, but the expression should still be incorporated as part of a complete statement.

As a model for what I am looking for, I have started the first answer for you.

- i.) Suppose X and Y are sets. What does it mean for Y to be a *subset* of X ?

The set Y is a subset of X if ...

every element of Y is an element of X .

- ii.) Suppose X is a set. What is the *power set* of X ?

The power set of X is the set of all subsets of X . In set builder notation,

$$\mathcal{P}(X) = \{A : A \subseteq X\}.$$

- iii.) Suppose X and Y are sets. What is the *Cartesian product* of X and Y (in that order)?

The Cartesian product $X \times Y$ is the set of ordered pairs where the first coordinate is an element of X and the second is an element of Y . That is,

$$X \times Y = \{(a, b) : a \in X \text{ and } b \in Y\}.$$

3. (4 pts.) Suppose A is a finite set of cardinality n . Compute each of the following.

(a) $|A^4| = n^4$

(b) $|\mathcal{P}(A)| = 2^n$

(c) $|\mathcal{P}(A^2)| = 2^{n^2}$

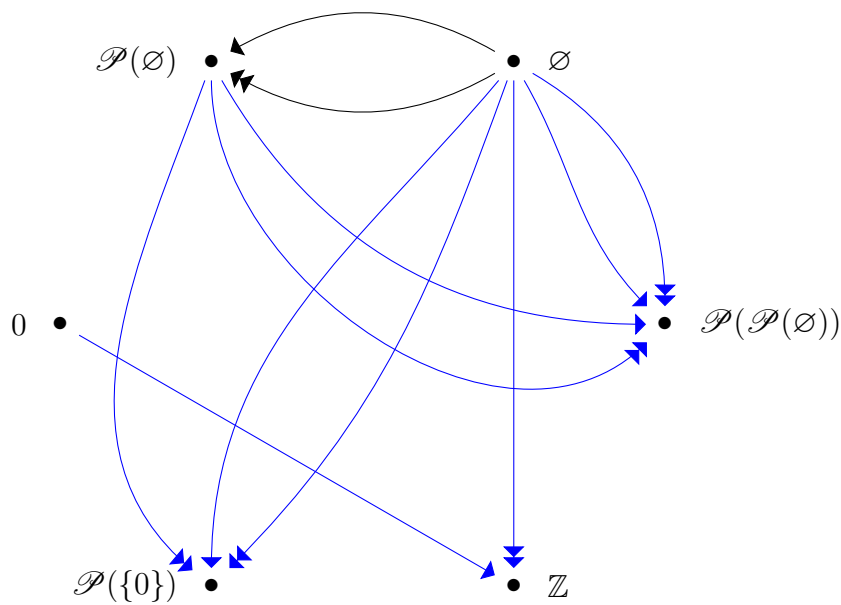
(d) $|A^2 \times \mathcal{P}(\emptyset)| = n^2$

(Continued on reverse.)

4. (1 pt per correct arrow, -0.5 pt per incorrect arrow)

- Draw a **single-headed arrow** from one set to the next if the first is an **element** of the second.
- Draw a **double-headed arrow** if the first element is a **subset** of the second.

As an example, I have drawn two arrows for you.



It can be helpful to write out some of these sets:

$$\mathcal{P}(\emptyset) = \{\emptyset\}$$

$$\mathcal{P}(\mathcal{P}(\emptyset)) = \{\emptyset, \{\emptyset\}\} = \{\emptyset, \mathcal{P}(\emptyset)\}$$

$$\mathcal{P}(\{0\}) = \{\emptyset, \{0\}\}.$$