MATH 2001
QUIZ 7

Work in groups of up to three people to give a complete proof of the following statement.

Problem. Prove that $\{18n + 8m : n \in \mathbb{Z} \text{ and } m \in \mathbb{Z}\}$ is equal to the set of even integers.

Points are awarded as follows:

- (1 pt) Name(s) in the top right corner.
- (1 pt) Writing is neat and legible.
- (1 pt) Each relevant and correctly stated definition.
- (7 pts) Complete sketch of the proof.
- (7 pts) Complete proof: introductory statements, all statements are complete sentences, every statement is justified appropriately, etc.

You may use the space below for scratch work, write your outline and proof on separate sheets.

Date: June 11, 2016.

MATH 2001
QUIZ 7 – Outline

1. Write your name(s) in the top right corner.
2. Write neatly and legibly.
3. State the definitions cited in your proof.
4. Provide a complete outline for a proof of the following claim.

Problem. Prove that $\{18n + 8m : n \in \mathbb{Z} \text{ and } m \in \mathbb{Z}\}$ is equal to the set of even integers.

Definitions:

Definition (Even)**.** An integer a is *even* if $a = 2c$ for some $c \in \mathbb{Z}$.

Theorem (Double containment)**.** *If A and B are sets, then $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.*

Definition (Subset)**.** If A and B are sets, then $A \subseteq B$ if $x \in A \Rightarrow x \in B$.

Outline:

Let $A = \{18n + 8m : n \in \mathbb{Z} \text{ and } m \in \mathbb{Z}\}$ and $B = \{2c : c \in \mathbb{Z}\}$.

To prove that $A = B$, prove that $A \subseteq B$ and $B \subseteq A$.

($A \subseteq B$):

$$\begin{aligned} x \in A &\Rightarrow x = 18n + 8m \quad \text{for some } n, m \in \mathbb{Z} \\ &\Rightarrow x = 2(9n + 4m) \\ &\Rightarrow x = 2c \quad \text{where } c = 9n + 4m \in \mathbb{Z} \\ &\Rightarrow x \in B. \end{aligned}$$

($B \subseteq A$):

$$\begin{aligned} x \in B &\Rightarrow x = 2c \quad \text{for some } c \in \mathbb{Z} \\ &\Rightarrow x = (18 + 8(-2))c \\ &\Rightarrow x = 18c + 8(-2c) \\ &\Rightarrow x = 18n + 8n \quad \text{where } n = c \in \mathbb{Z} \text{ and } m = -2c \in \mathbb{Z} \\ &\Rightarrow x \in A. \end{aligned}$$

MATH 2001
QUIZ 7 – Proof

1. Write your name(s) in the top right corner.
2. Write is neatly and legibly.
3. Provide a complete proof of the following claim.

Problem. Prove that $\{18n + 8m : n \in \mathbb{Z} \text{ and } m \in \mathbb{Z}\}$ is equal to the set of even integers.

Proof. Let $A = \{18n + 8m : n \in \mathbb{Z} \text{ and } m \in \mathbb{Z}\}$, and let B be the set of even integers. We prove that $A = B$ by showing that $A \subseteq B$ and $B \subseteq A$.

($\subseteq$) We start by proving that $A \subseteq B$ by showing that if $x \in A$, then $x \in B$.

Suppose that $x \in A$. Then $x = 18n + 8m$ for some $n, m \in \mathbb{Z}$. Hence $x = 2(9n + 4m) = 2c$, where $c = 9n + 4m \in \mathbb{Z}$, and thus x is even by definition. Therefore, $x \in B$, and thus $A \subseteq B$ by the definition of subset.

($\supseteq$) We now prove that $B \subseteq A$ by showing that if $x \in B$, then $x \in A$.

Suppose that $x \in B$, that is, suppose that x is even. Then $x = 2c$ for some $c \in \mathbb{Z}$. Note that

$$\begin{aligned} x &= 2c \\ &= (18 + 8(-2))c \\ &= 18c + 8(-2c) \\ &= 18n + 8m, \end{aligned}$$

where $n = c$ and $m = -2c$. Since c is an integer, so are m and n, hence $x \in A$. Thus, by the definition of subset, $B \subseteq A$.

As $A \subseteq B$ and $B \subseteq A$, we have proven that $A = B$. $\square$