# MATH 2001
# QUIZ 8

On this quiz, you are tasked to analyze and assess a proof.

### INSTRUCTIONS

**Task 1.** [First glance.] Give the argument a *quick* once-over. On this first pass, you do **not** need to understand the proof. **Do not get caught up in the details.**

For each sentence, ask the question: "do I understand, or could I work out, the meaning of this statement?"

- If 'yes' ("this statement seems reasonable enough"), put a ✓ by the sentence number.
- If 'no' ("What? Something seems wrong. I'm confused."), put a **?** above the point(s) in the sentence that you find confusing, and **move on** to the next sentence.

**Task 2.** [Understand.] Read through the proof a second time with an eye towards understanding the argument as a whole.

Confusion often arrises from poorly worded sentences or inadequately justified statements.

- Attempt to resolve the confusions from the first reading (though it is fine if you cannot).

  Make small edits to the proof to aid your understanding of what is written.

(You may be tempted to rewrite the whole sentence, or even the entire argument. In the interest of time, I encourage you to not do this.)

**Task 3.** [What is being proved?] In the proof, the statement that the author is proving is not made explicit. We would like to recover that statement: "Given ____________, prove that ____________."

- Circle any information which you believe to be given in the statement of the problem and mark it with a "G".
- The author claims that they will show "if $Z \in C$ and $Z \in$ A." What will they have proven if they achieve this statement?

  Using the answers to these last two questions, write a statement of the form

  "Given ____________, prove that ____________"

  above the proof.

**Task 4.** [Is the argument sound?]

- Is the argument logically sound? You may want to sketch an outline of the proof.

**Task 5.** [Grade]

- At the bottom of the proof, give a grade based solely on the quality of writing.
- At the bottom of the proof, give a grade based solely on the logical accuracy of argument.

---

*Date*: March 17, 2016.

# Quiz 8 proof

*Proof.* [1] A set is a collection, of some elements, and so are $B$ and $C$ . [2] Set $C = \overline{B}$ where by definition $\overline{B} = \{X \notin B\}$ and $B = \overline{A}$ where $\overline{A} = \{Y \notin A\}$ both by the definition of complementary set. [3] Set $Z \in C$. [4] We prove the result by showing if $Z \in C$ and $Z \in \mathrm{A}$. [5] Since $Z \in \overline{B}, Z \notin B$ by definition and because $C = \overline{B}$. [6] On the the other hand since $Y \notin A$ , $Y \in \overline{A}$, and $Y \in B$ because $B$ and $\overline{A}$ are the same set. [7] Therefore if $Z \notin Y$, than $Z \notin B$, $Z \notin \overline{\mathrm{A}}$ and $Z \in \mathrm{A}$. [8] Thus $Z \in C$ and $Z \in \mathrm{A}$. [9] Proof over. □

Some questions to consider (but you don't need to answer).

a.) In sentence 2, is the first word ("set") a noun or a verb?

b.) In sentence 3, is $Z$ a set, a subset, an element, or some combination of these (which ones)?

c.) In sentence 5, what is the definition to which the author refers?

d.) There are $A$'s and there are A's. Did you consider the possibility that these were two different sets?

There isn't a solution to this quiz per say, so I will "simply" offer my comments.

Billiam's proof

The name of the author has been changed so as to protect their identity.

*Proof.* [1] A set is a collection, of some elements, and so are $B$ and $C$ .

This sentence is silly; I almost don't know what to say.

- The first comma is wrong. [remove]
- There is a space before the period. [remove]
- The sentences starts out as the definition of a set... and then $B$ and $C$ are also sets? The confusion is compounded by the fact that Billiam never formally declares that $A$ (or A?) is a set, but maybe that's what they are doing at the start of this sentence: "A is a set, and so are $B$ and $C$"? And somehow Billiam is also trying to jam the definition into this sentence? In any case, this is a poor way of saying, "let ($A$?) $B$ and $C$ be sets." [rewrite]

[2] Set $C = \overline{B}$ where by definition $\overline{B} = \{X \notin B\}$ and $B = \overline{A}$ where $\overline{A} = \{Y \notin A\}$ both by the definition of complementary set.

This is a common issue: stating a definition or explaining notation in the midst of other details. The sentence is cumbersome and it becomes difficult to pick out the important information. Although it is often helpful to remind the reader of the meaning of a term, clunky sentences detract from the argument. In general, it is a much better practice to remind the reader of the terms elsewhere.

Suggestions:

"Let $C = \overline{B}$ and $B = \overline{A}$." (Just give the important information.)

"Let $C = \overline{B}$ and $B = \overline{A}$, where the bar denotes the complement of the set." (A friendly reminder of the notation in a way that doesn't disrupt the important information. The full definition of complement can be given elsewhere.)

- This sentence is poorly written for the reason listed above. [rewrite]
- The set notation is incorrect. This is neither set-builder notation, nor is it an explicit presentation of elements. [fix set notation]
- I don't think I've ever heard "complementary" used for sets (unlike *complementary angles* in geometry, say). "The set $B$ is the complement of $C$", or "$B$ and $C$ are complements" is much more standard. [change to "definition of set complement"]

[3] Set $Z \in C$.

This is fine, though "suppose $Z \in C$" or "let $Z \in C$" are preferable due to the potential noun/verb ambiguity of "set".

[4] We prove the result by showing if $Z \in C$ and $Z \in$A.

Stating the 'result' would be helpful. Perhaps Billiam misunderstood the question and his method doesn't actually address the original question.

- "showing if $Z \in C$ and $Z \in A$" is a fragment: "showing if [this] and [that], then...". Or maybe this is just mis-worded. Without knowing the result, I don't know. [complete or reword sentence]
- TeX error at the end of line. [fix]

[5] Since $Z \in \overline{B}, Z \notin B$ by definition and because $C = \overline{B}$.

- Why is $Z \in \overline{B}$?

  This is an example of a disconnected thought as this line follows directly from the third: if $Z \in C$, then $Z \in \overline{B}$ because $C = \overline{B}$. The fourth sentence is out of place and breaks the logical flow of the argument. [swap sentences 3 and 4]

- What definition?

  Probably the definition of complement, but we have a lot of definitions. It's better to be explicit and avoid ambiguity.

- Is the comma an 'and' or a 'then'?

  Not such a big deal on this line, but it is a bit ambiguous: "since $Z \in \overline{B}$ and $Z \notin B$..." or "since $Z \in \overline{B}$, then $Z \notin B$..." [insert helping word]

[6] On the the other hand since $Y \notin A$ , $Y \in \overline{A}$, and $Y \in B$ because $B$ and $\overline{A}$ are the same set.

- Another common mistake: this is another big jump in logic. We went from $Z \notin B$ to $Y \notin A$ without explanation. Do not do this in your proofs.

  Consider if you did this with directions.

  > Okay Zorro, I'm going to tell you how to get from Denver to New York City. If you're in Denver, you can take I-70 to St. Louis. On the other hand, Yvonne is in New York City, and she can take the NJ Turnpike down to Philadelphia.

  What? What does Yvonne have to do with Zorro's mission? Keep talking to Zorro!

  [remove this sentence]

- Here's where the ambiguous comma issue really shows. If you don't want it to look like a list, put some words between the commas.
- Extra space before first comma.
- "The" is written twice at the start of the sentence.

[7] Therefore if $Z \notin Y$, than $Z \notin B$, $Z \notin \overline{A}$ and $Z \in A$.

- How can we determine if $Z$ is or is not an element of $Y$? Probably $Z \neq Y$ is more appropriate since it appears that we are comparing elements in the sets $A$, $B$, and $C$. Even so...
- There are definite logical gaps here. Basically any justification for "if $Z \notin Y$, then $Z \notin B$" is wrong.
  - If $Z \notin Y$, then $Z \notin B$ only if $Y \subseteq B$. But we don't know whether $Y \subseteq B$ or not. So that's out.
  - If $Z \neq Y$, then it still could be that $Z \in B$.

  Oh wait, we already established that $Z \notin B$ back in sentence 5. Again I ask: why is Yvonne in this story?

- Commas are fine here. This is actually a list of things that we can conclude. Justification would be nice though. [justify statements]
- 'than' is the wrong word. [fix]

[8] Thus $Z \in C$ and $Z \in A$.

Okay. At least we ended where Billiam said we would.

[9] Proof over.

This is a fragment. Also, if you follow the (now archaic? (too soon to call it 'archaic'?)) rule against ending a sentence with a preposition, there's that as well. (I guess I'm archaic.) [fix]

□

Some questions to consider (but you don't need to answer).

a.) In sentence 2, is the first word ("set") a noun or a verb?

To me, it is a verb, but there is space for ambiguity.

b.) In sentence 3, is $Z$ a set, a subset, an element, or some combination of these (which ones)?

Again, 'set' better be a verb (if not, this is a terribly worded sentence), so the only thing that this line is telling us is that $Z$ is an element of the set $C$.

c.) In sentence 5, what is the definition to which the author refers?

Already addressed.

d.) There are $A$'s and there are A's. Did you consider the possibility that these were two different sets?

No.

## Summary

Billiam argued that if $Z \in C$, then $Z \in C$ and $Z \in A$, which answers the following prompt.

Given the sets $A$, $B$, and $C$, where $C = \overline{B}$ and $B = \overline{A}$, prove that $C \subseteq C \cap A$.

Admittedly, it is very odd to start with the assumption that $Z \in C$ and then conclude that $Z \in C$.

## Grades

**Quality of writing:** Let's do a quick rundown. Out of the nine sentences, 3 and 8 are perfectly fine. Sentences 1, 2, and 6 are cumbersome head-scratchers, and lines 4 and 9 suffer from the grammatical deficiency of not being a sentence. That leaves 7, which has a typo, but otherwise works.

I give this one 6 fleebles.

**Quality of logic** Sentences 1: meh? Sentence 2 has set notation issues. Lines 3 and 8 are good. I have a tiff with 4, a spat with 5, a quarrel with 7, and 6 should be burned. Line 9: n/a.

The ideas are here and they are mostly in order, but they are interspersed with unnecessary detours. Not a great sign for demonstrating understanding of the subject. All in all, not too bad. Easy fix: cut out the bad parts.

Award: 7 schmeckles.