MATH 2001
DEFINITIONS: "FOR SOME"

Definition (Divides)**.** If $a, b \in \mathbb{Z}$, then a *divides* b (denoted $a \mid b$) if $ac = b$ for some $c \in \mathbb{Z}$.

If $a \mid b$, then a is a *divisor* of b, and b is a *multiple* of a.

Definition (Even)**.** If $a \in \mathbb{Z}$, then a is *even* if $a = 2c$ for some $c \in \mathbb{Z}$.

Definition (Odd)**.** If $a \in \mathbb{Z}$, then a is *odd* if $a = 2c + 1$ for some $c \in \mathbb{Z}$.

When we say "for some $c \in \mathbb{Z}$," we mean that

1. there exists at least one value that satisfies the condition, and
2. we let c represent one of those values.

Once the value is named, we can use c as if it is the actual thing, and not just some arbitrary integer.

For example, *someone* is in control of this classroom. Let us call that person ♩♫ *Mr. F* ♩♫. Now, we can refer to Mr. F (♩♫) as if he is an actual person, even if we do not know his identity.

"Rita claims Mr. F is her uncle!" not "Rita claims Mr. F is her uncle for some person Mr. F!"

Similarly, saying $ac = b$ for some $c \in \mathbb{Z}$ means that

"c is a solution to $ax = b$," not "c is a solution to $ax = b$ for some $c \in \mathbb{Z}$."

Exercise 1. Prove that the sum of two odd integers is even.

Exercise 2. Prove that $\{8^n : n \in \mathbb{Q}\} = \{2^n : n \in \mathbb{Q}\}$.

Exercise 3. Prove that the product of an even integer with any other integer is even.

Exercise 4. Prove that if a is odd, then $8 \mid (a^2 - 1)$.

Upcoming deadlines:

- Due Wednesday, Feb 24: second draft of proof 3, and first draft of proof 5.
- Due Friday, Feb 26: final draft of proof 2, second draft of proof 4.
- Due Monday, Feb 29: final draft of proof 3, second draft of proof 5, first draft of proof 6.

As the number of proofs are piling up, from proof 6 onwards, I will only be giving one round of comments before final copies are due.