MATH 2001
STATEMENTS AND NEGATION

Simple statements: "P". A statement (generally denoted P) is an expression that is decidedly true or false. By negating a statement ($\neg P$), we change its meaning from true to false, or false to true. A negation generally involves inserting or removing 'not' from the statement, though that is not always the case.

Exercise 1. Negate each of the following statements.

P	$\neg P$
I went to the store.	
No parking on week days.	
$\pi \in \mathbb{Z}$.	
$2 + 3 > 6$.	

And/or statements: "P and/or Q". A simply way to combine statements is to use the 'and' or 'or' conjunction. What happens when you negate such a statement?

Exercise 2. Negate each of the following statements.

P and/or Q	$\neg(P$ and/or $Q)$
I am 33 years old or I am 34 years old.	
3 is positive, but 4 is not.	
$\pi \in \mathbb{Q}$ and $\pi \notin \mathbb{Q}$.	
$2 + 3 > 6$ or $2 + 3 < 0$.	

Exercise 3. As a general rule:

- $\neg(P$ and $Q) =$
- $\neg(P$ or $Q) =$

(If you are familiar with the logical operators $\wedge$ and $\vee$, feel free to use them here.)

If-then statements: "if P, then Q". Perhaps the most common form of a statement in mathematics is the if-then statement. Since if-then statements are implications, the statement "if P, then Q" is equivalent to the statement "$P \Rightarrow Q$". The negation of an if-then statement is given by the following rule:

- $\neg(P \Rightarrow Q) = (P$ and $\neg Q)$.

Exercise 4. Negate each of the following statements. For each statement, indicate whether the statement is true or false.

$P \Rightarrow Q$	$\neg(P \Rightarrow Q)$
If it is Monday, then we have class.	
The light is green, so we can go.	
$x^2 \in \mathbb{Z} \Rightarrow x \in \mathbb{Z}$.	
If x^2 is odd, then x is odd.	

Converse. The *converse* of $P \Rightarrow Q$ is $Q \Rightarrow P$.

Exercise 5. Write the converse of each of the following statements.

$P \Rightarrow Q$	$Q \Rightarrow P$
If it is Monday, then we have class.	
The light is green, so we can go.	
$x^2 \in \mathbb{Z} \Rightarrow x \in \mathbb{Z}$.	
If x^2 is odd, then x is odd.	

How is a statement related to its converse? Are they equivalent? Are they negations of each other? Or are they unrelated?

Contrapositive. The *contrapositive* of $P \Rightarrow Q$ is $\neg Q \Rightarrow \neg P$.

Exercise 6. Write the contrapositive of each of the following statements.

$P \Rightarrow Q$	$\neg Q \Rightarrow \neg P$
If it is Monday, then we have class.	
The light is green, so we can go.	
$x^2 \in \mathbb{Z} \Rightarrow x \in \mathbb{Z}$.	
If x^2 is odd, then x is odd.	

How is a statement related to its contrapositive? Are they equivalent? Are they negations of each other? Or are they unrelated?

Upcoming deadlines:

- Due Monday, Feb 29: final draft of proof 3, second draft of proof 5, first draft of proof 6.
- Due Wednesday, Mar 2: final draft of proof 4, first draft of proof 7.
- Due Friday Mar 4: final draft of proof 5, final draft of proof 6.

As the number of proofs are piling up, from proof 6 onwards, I will only be giving one round of comments before final copies are due.