MATH 2001
OPEN AND CLOSED SETS

Upcoming deadlines:

- Due Friday, Mar 18: final draft of proof 9.
- Due Wednesday, Mar 30: final draft of proof 10, first draft of proof 11.

Definition. Let A be a set, let $\mathcal{B}$ be a basis for a topology on A, and let X be a subset of A. The set X is *open* if for each $x \in X$, there exists a $B \in \mathcal{B}$ such that $x \in B$ and $B \subseteq X$.

Definition. Let $\mathcal{B}$ be a basis for a topology on A, and let X be a subset of A. The set X is *closed* if $A - X$ (the complement of X in A) is open.

Exercise 1. Let $\mathcal{B} = \{(x - \epsilon, x + \epsilon) : \epsilon \in \mathbb{R}, \epsilon > 0\}$. (The topology which this basis generates is known as the *standard topology on* $\mathbb{R}$.) For each of the following sets, determine whether the set is open, closed, both, or neither. (Assume $a, b \in \mathbb{R}$.)

a. (a, b)

b. $[a, b)$

c. $(-\infty, b]$

d. $[a, b]$

e. $\mathbb{Z}$

f. $\mathbb{R}$

g. $\varnothing$

h. $\{10^{-n} : n \in \mathbb{N}\}$

Exercise 2. Let $\mathcal{B} = \{U \subseteq \mathbb{R} : |\overline{U}| < \infty\}$. (This is a basis for the *finite complement topology on* $\mathbb{R}$.) Repeat the previous exercise with this topology.

Exercise 3. Let $\mathcal{B} = \{[x, y] \subseteq \mathbb{R} : x, y \in \mathbb{R}\}$. (This basis for the *discrete topology on* $\mathbb{R}$.) Repeat the previous exercise with this topology.