MATH 2001
EQUIVALENCE RELATIONS

Homework. Book exercises: Due Wednesday, April 13.
Section 11.0: 5, 7.
Section 11.1: 2, 8.
Section 11.2: 2, 4, 8.
Section 11.3: 4.

Proofs.
Friday, April 8: first draft of Proof 13.
Monday, April 11: final draft of Proof 12.
Wednesday, April 13: final draft of Proof 13 and first draft of Proof 14.

Example 1. Let $A = \{1, 2, 3, 4, 5, 6\}$, and let R be an equivalence relation on A defined by

$$R = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6), (2, 3), (3, 2), (4, 6), (6, 4), (4, 5), (5, 4), (5, 6), (6, 5)\}.$$

List the equivalence classes of R.

Example 2. Let R be an equivalence relation on A, where $A = \{a, b, c, d, e\}$. Suppose that $(a, d) \in R$ and $(b, c) \in R$. Write out the elements of R, and draw the graph of R.

Example 3. Let R be the relation on $\mathbb{Z}$ defined by

$$R = \{(a, b) : a, b \in \mathbb{Z}, 3a - 5b \text{ is even}\}.$$

Describe the equivalence classes.

Theorem 4. *Suppose that R is an equivalence relation on a set A, and suppose also that $a, b \in A$. Then $[a] = [b]$ if and only if $(a, b) \in R$.*

Proof.

□

Definition 5. A *partition* of a set A is a set of non-empty subsets of A such that the union of all the subsets is equal to A, and the sets are pairwise disjoint. That is, if X and Y are in the partition, then $X \cap Y = \varnothing$.

Example 6. Find all the partitions of $A = \{a, b, c\}$.

Theorem 7. *Let R be an equivalence relation on A. Then $\{[a] : a \in A\}$ is a partition of A.*