## MATH 2001 EQUIVALENCE RELATIONS

Homework. Book exercises: Due Wednesday, April 13.

Section 11.0: 5, 7.

Section 11.1: 2, 8.

Section 11.2: 2, 4, 8.

Section 11.3: 4.

## Proofs.

Friday, April 8: first draft of Proof 13.

Monday, April 11: final draft of Proof 12.

Wednesday, April 13: final draft of Proof 13 and first draft of Proof 14.

**Example 1.** Let  $A = \{1, 2, 3, 4, 5, 6\}$ , and let R be an equivalence relation on A defined by  $R = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6), (2, 3), (3, 2), (4, 6), (6, 4), (4, 5), (5, 4), (5, 6), (6, 5)\}.$ 

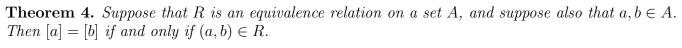
List the equivalence classes of R.

**Example 2.** Let R be an equivalence relation on A, where  $A = \{a, b, c, d, e\}$ . Suppose that  $(a, d) \in R$  and  $(b, c) \in R$ . Write out the elements of R, and draw the graph of R.

**Example 3.** Let R be the relation on  $\mathbb{Z}$  defined by

$$R = \{(a, b) : a, b \in \mathbb{Z}, 3a - 5b \text{ is even}\}.$$

Describe the equivalence classes.



Proof.

**Definition 5.** A partition of a set A is a set of non-empty subsets of A such that the union of all the subsets is equal to A, and the sets are pairwise disjoint. That is, if X and Y are in the partition, then  $X \cap Y = \emptyset$ .

**Example 6.** Find all the partitions of  $A = \{a, b, c\}$ .

**Theorem 7.** Let R be an equivalence relation on A. Then  $\{[a] : a \in A\}$  is a partition of A.