MATH 2001
INTRODUCTION TO CARTESIAN PRODUCT

Exercise 1. What is a *Cartesian product*? State the exact definition.

Exercise 2. Let $A = \{0, 1\}$ and $B = \{\varnothing\}$. Write out the sets A^2 and $B \times A$ explicitly.

Exercise 3. Let $A = \{a, b, \{a, b\}, (a, b), \mathbb{Z}\}$. True or false?

T   **F** : $\varnothing \in A$ **T**   **F** : $\varnothing \subseteq A$

T   **F** : $\varnothing \in \mathscr{P}(A)$ **T**   **F** : $\varnothing \subseteq \mathscr{P}(A)$

T   **F** : $\{a, b\} \in A$ **T**   **F** : $\{a, b\} \subseteq A$

T   **F** : $\{a, b\} \in \mathscr{P}(A)$ **T**   **F** : $\{a, b\} \subseteq \mathscr{P}(A)$

T   **F** : $(a, b) \in A^2$ **T**   **F** : $(a, b) \subseteq A^2$

T   **F** : $\{(a, a)\} \in A^2$ **T**   **F** : $\{(a, a)\} \subseteq A^2$

T   **F** : $\{a, b\} \in A^2$ **T**   **F** : $\{a, b\} \subseteq A^2$

T   **F** : $1 \in A$ **T**   **F** : $1 \subseteq A$

Exercise 4. Let $A = \{0, 1\}$, $B = \{\varnothing\}$, and $C = A \times B$. Consider the following sets:

- $A \times A \times B$
- $A \times C$
- $A^2 \times B$
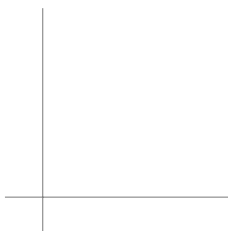- $A \times (A \times B)$
- $(A \times A) \times B$

What are the distinctions between these sets? Are all/any of them the same? If not, how do the sets differ?

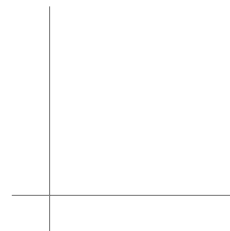**Exercise 5.** Sketch each of the following sets in the x, y-plane.
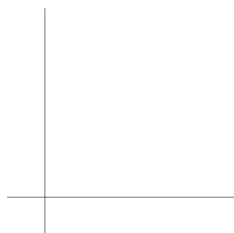
a. $[1, 2]^2$

c. $[1, 2] \times \{1, 2\}$
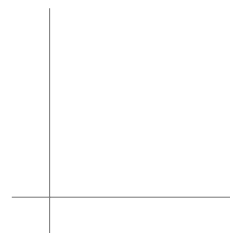
e. $\mathbb{R} \times \{1, 2\}$

b. $\{1, 2\}^2$

d. $\mathbb{R} \times \mathbb{N}$

f. $(0, 1) \times \mathbb{R}$

Exercise 6. Suppose A and B are finite sets. What is the cardinality of $A \times B$? In a few sentences, justify your claim.

Homework. Due Friday, January 29 at 2pm.

- Read Section 1.2 from the text.

- Complete the following exercises (add these to your Overleaf file with the other book problems).

    – Section 1.2: 2a, 2b, 6