## **MATH 2001 DEFINITIONS: REVIEW**

**Definition 1** (Set equality). If A and B are sets, then A = B if  $x \in A \Rightarrow x \in B$  and  $x \in B \Rightarrow x \in A$ .

**Definition 2** (Subset). If A and B are sets, then  $A \subseteq B$  if  $x \in A \Rightarrow x \in B$ .

**Definition 3** (Power set). If A is a set, then  $\mathscr{P}(A) = \{x : x \subseteq A\}$ .

**Definition 4** (Union). If A and B are sets, then  $A \cup B = \{x : x \in A \text{ or } x \in B\}$ .

**Definition 5** (Finite and infinite unions). If  $\{A_i\}$  is a collection of sets indexed by I, then

$$\bigcup_{i \in I} A_i = \{x : x \in A_i \text{ for some } i \in I\}.$$

**Definition 6** (Intersection). If A and B are sets, then  $A \cap B = \{x : x \in A \text{ and } x \in B\}$ .

**Definition 7** (Finite and infinite intersection). If  $\{A_i\}$  is a collection of sets indexed by I, then

$$\bigcap_{i \in I} A_i = \{x : x \in A_i \text{ for every } i \in I\}.$$

**Definition 8** (Set difference). If A and B are sets, then  $A - B = \{x : x \in A \text{ and } x \notin B\}$ .

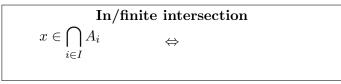
**Definition 9** (Complement). If A is a set, then  $\overline{A} = \{x : x \notin A\}$ .

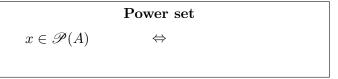
**Exercise 1.** Fill out the right side of each block.

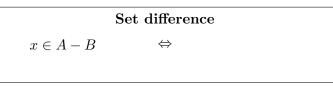
	Set equality					
A = B	$\Leftrightarrow$					

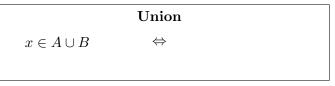
Set equality		Intersection
$\Leftrightarrow$	$x \in A \cap B$	$\Leftrightarrow$













$$x \in \bigcup_{i \in I} A_i \qquad \Leftrightarrow \qquad \qquad \Leftrightarrow$$

All of the proof that we have been writing recently have been of the form, "prove that  $A \subseteq B$ ." In order to prove that  $A \subseteq B$ , we show that A and B satisfy the definition of subset. Namely, we show that if  $x \in A$ , then  $x \in B$ . In particular, the body of the proof should start with the statement, "suppose  $x \in A$ " (or "if  $x \in A$ "). Then after a series of logical deductions, the proof ends once we conclude that  $x \in B$ .

**Exercise 2.** Arrange the following statements to give an outline for a proof that  $(A \cap B) - C \subseteq (A - C) \cap (B - C)$ . Justify each statement by citing the appropriate definition.

**Exercise 3.** Since unions are 'or' statements, proofs involving unions often break into multiple cases. Arrange the statements to prove that  $(A-C) \cup (B-C) \subseteq (A \cup B) - C$ . Justify each statement by citing the appropriate definition.

	<i>&gt;</i>	(by definition of	)	
Case 1:	<i>⇒</i>	(by definition of	)	a. $x \in A - C$ b. $x \in B - C$
	⇒	(by definition of	)	c. $x \in B - C$
	⇒	(by definition of	)	d. $x \in (A \cup B) - C$
	⇒	(by definition of	)	e. $x \in B$ and $x \notin C$
Case 2:	<i>&gt;</i>	(by definition of	)	f. $x \in A$ and $x \notin C$ g. $x \in A$ or $x \in B$ , and $x \notin C$
	⇒	(by definition of	)	h. $x \in A - C$ or $x \in B - C$
	⇒	(by definition of	)	i. $x \in (A-C) \cup (B-C)$
	⇒	(by definition of	)	

Exercise 4. In a similar fashion, sketch proofs for the following statements. In some cases, the justification for a step might not be a definition, but information that was given in the statement of the problem.

- a. Prove that if  $X \subseteq A \cap B$ , then  $X \subseteq A$  and  $X \subseteq B$ .
- b. Prove that  $\mathscr{P}(A \cap B) \subseteq \mathscr{P}(A) \cap \mathscr{P}(B)$ .
- c. Prove that if  $\mathscr{P}(A) \subseteq \mathscr{P}(B)$ , then  $A \subseteq B$ .

## Upcoming deadlines:

- Due Friday, Feb 19: first draft of Proof 4.
- Due Monday, Feb 22: final draft of Proof 1 and second draft of proof 2.
- Due Wednesday, Feb 24: second draft of proof 3.