MATH 2001
DEFINITIONS: REVIEW

Definition 1 (Set equality)**.** If A and B are sets, then $A = B$ if $x \in A \Rightarrow x \in B$ and $x \in B \Rightarrow x \in A$.

Definition 2 (Subset)**.** If A and B are sets, then $A \subseteq B$ if $x \in A \Rightarrow x \in B$.

Definition 3 (Power set)**.** If A is a set, then $\mathscr{P}(A) = \{x : x \subseteq A\}$.

Definition 4 (Union)**.** If A and B are sets, then $A \cup B = \{x : x \in A \text{ or } x \in B\}$.

Definition 5 (Finite and infinite unions)**.** If $\{A_i\}$ is a collection of sets indexed by I, then

$$\bigcup_{i \in I} A_i = \{x : x \in A_i \text{ for some } i \in I\}.$$

Definition 6 (Intersection)**.** If A and B are sets, then $A \cap B = \{x : x \in A \text{ and } x \in B\}$.

Definition 7 (Finite and infinite intersection)**.** If $\{A_i\}$ is a collection of sets indexed by I, then

$$\bigcap_{i \in I} A_i = \{x : x \in A_i \text{ for every } i \in I\}.$$

Definition 8 (Set difference)**.** If A and B are sets, then $A - B = \{x : x \in A \text{ and } x \notin B\}$.

Definition 9 (Complement)**.** If A is a set, then $\overline{A} = \{x : x \notin A\}$.

Exercise 1. Fill out the right side of each block.

Set equality	
$A = B$	$\Leftrightarrow$

Subset	
$A \subseteq B$	$\Leftrightarrow$

Power set	
$x \in \mathscr{P}(A)$	$\Leftrightarrow$

Union	
$x \in A \cup B$	$\Leftrightarrow$

In/finite union	
$x \in \bigcup_{i \in I} A_i$	$\Leftrightarrow$

Intersection	
$x \in A \cap B$	$\Leftrightarrow$

In/finite intersection	
$x \in \bigcap_{i \in I} A_i$	$\Leftrightarrow$

Set difference	
$x \in A - B$	$\Leftrightarrow$

Complement	
$x \in \overline{A}$	$\Leftrightarrow$

All of the proof that we have been writing recently have been of the form, "prove that $A \subseteq B$." In order to prove that $A \subseteq B$, we show that A and B satisfy the definition of subset. Namely, we show that if $x \in A$, then $x \in B$. In particular, the body of the proof should start with the statement, "suppose $x \in A$" (or "if $x \in A$"). Then after a series of logical deductions, the proof ends once we conclude that $x \in B$.

Exercise 2. Arrange the following statements to give an outline for a proof that $(A \cap B) - C \subseteq (A - C) \cap (B - C)$. Justify each statement by citing the appropriate definition.

_____ $\Rightarrow$ _____	(by definition of)	a. $x \in A$ and $x \in B$ and $x \notin C$
$\Rightarrow$ _____	(by definition of)	b. $x \in A \cap B$ and $x \notin C$
$\Rightarrow$ _____	(by definition of)	c. $x \in (A \cap B) - C$
$\Rightarrow$ _____	(by definition of)	d. $x \in A - C$ and $x \in B - C$
		e. $x \in (A - C) \cap (B - C)$

Exercise 3. Since unions are 'or' statements, proofs involving unions often break into multiple cases. Arrange the statements to prove that $(A - C) \cup (B - C) \subseteq (A \cup B) - C$. Justify each statement by citing the appropriate definition.

_____ $\Rightarrow$ _____	(by definition of)	
Case 1: _____ $\Rightarrow$ _____	(by definition of)	a. $x \in A - C$
$\Rightarrow$ _____	(by definition of)	b. $x \in B - C$
$\Rightarrow$ _____	(by definition of)	c. $x \in A \cup B$ and $x \notin C$
$\Rightarrow$ _____	(by definition of)	d. $x \in (A \cup B) - C$
Case 2: _____ $\Rightarrow$ _____	(by definition of)	e. $x \in B$ and $x \notin C$
$\Rightarrow$ _____	(by definition of)	f. $x \in A$ and $x \notin C$
$\Rightarrow$ _____	(by definition of)	g. $x \in A$ or $x \in B$, and $x \notin C$
$\Rightarrow$ _____	(by definition of)	h. $x \in A - C$ or $x \in B - C$
		i. $x \in (A - C) \cup (B - C)$

Exercise 4. In a similar fashion, sketch proofs for the following statements. In some cases, the justification for a step might not be a definition, but information that was given in the statement of the problem.

a. Prove that if $X \subseteq A \cap B$, then $X \subseteq A$ and $X \subseteq B$.

b. Prove that $\mathscr{P}(A \cap B) \subseteq \mathscr{P}(A) \cap \mathscr{P}(B)$.

c. Prove that if $\mathscr{P}(A) \subseteq \mathscr{P}(B)$, then $A \subseteq B$.

Upcoming deadlines:

- Due Friday, Feb 19: first draft of Proof 4.
- Due Monday, Feb 22: final draft of Proof 1 and second draft of proof 2.
- Due Wednesday, Feb 24: second draft of proof 3.