MATH 2001
DEFINITIONS: REVIEW II

Definition 1 (Cartesian product)**.** If A and B are a sets, then $A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$.

Cartesian product		
$(a, b) \in A \times B$	$\Leftrightarrow$	

Exercise 1. Arrange the following statements to give an outline for a proof that $(A \cap B) \times C \subseteq (A \times C) \cap (B \times C)$. Justify each statement by citing the appropriate definition.

Proof. (Start by a short introduction defining the variables and describing what will be proved.)

_____ $\Rightarrow$ _____	(by definition of)	a. $x \in A$, $x \in B$, and $y \in C$.
$\Rightarrow$ _____	(by definition of)	b. $(x, y) \in (A \cap B) \times C$
$\Rightarrow$ _____	(by definition of)	c. $(x, y) \in A \times C$ and $(x, y) \in B \times C$
$\Rightarrow$ _____	(by definition of)	d. $x \in A \cap B$ and $y \in C$
		e. $(x, y) \in (A \times C) \cap (B \times C)$

$\square$

Exercise 2. In a similar fashion, sketch proof for the statement $(A \times C) \cap (B \times C) \subseteq (A \cap B) \times C$. Include a brief introduction for each proof.

In Proof 2 in the Proof Portfolio, I stated the following theorem.

Theorem. *If A and B are sets, then $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.*

This theorem (showing that two sets are subsets of each other) is the most common technique for proving that two sets are equal.

Exercise 3. Show that $A \times (B \cup C) = (A \times B) \cup (A \times C)$ by proving that

a. $A \times (B \cup C) \subseteq (A \times B) \cup (A \times C)$, and

b. $(A \times B) \cup (A \times C) \subseteq A \times (B \cup C)$.

Upcoming deadlines:

- Due Friday, Feb 19: first draft of Proof 4.
- Due Monday, Feb 22: final draft of Proof 1 and second draft of proof 2.
- Due Wednesday, Feb 24: second draft of proof 3, and first draft of proof 5.
- Due Friday, Feb 26: second draft of proof 4.