DEFINITION

Graph

DEFINITION

Adjacent

DEFINITION

Endpoint of an edge

DEFINITION

Incident

DEFINITION

Neighbors of a vertex

DEFINITION

Degree of a vertex

THEOREM

Graph relation

DEFINITION

Order of a graph

DEFINITION

Size of a graph

DEFINITION

Maximum and minimum degree

Let $G = (V, E)$ be a graph. If $u, v \in V$, then u is *adjacent* to v if $\{u, v\} \in E$. We also use the notation $u \sim v$ to denote that u is adjacent to v.

$u \sim v$	$\Leftrightarrow$	$\{u, v\} \in E$

Let $G = (V, E)$ be a graph. An vertex $v \in V$ is *incident* with the edge $e \in E$ if $v \in e$.

Let $G = (V, E)$ be a graph. The *degree* of a vertex $v \in V$ is the number of edges in G incident with v, and is denoted by

$$d(v) = \#\{e \in E : v \in e\}.$$

The *order* of a graph G is $|V(G)|$, the number vertices in G.

Let $G = (V, E)$ be a graph. The *maximum degree* and *minimum degree* of the vertices in G are

$$\Delta(G) = \max\{d(v) : v \in V\}$$
$$\delta(G) = \min\{d(v) : v \in V\}.$$

A *graph* is a pair $G = (V, E)$, where V is a nonempty finite set and E is a set of two-element subsets of V. The elements in V are *vertices* and the elements of E are *edges*.

Let $G = (V, E)$ is a graph. If $\{u, v\} \in E$, then the *endpoints* of $\{u, v\}$ are the vertices u and v.

Let $G = (V, E)$. For any $v \in V$, the *neighbors* of v is the set of vertices adjacent to v. The set of neighbors of v is denoted by

$$N(v) = \{u \in V : u \sim v\}.$$

For any graph $G = (V, E)$, we have

$$\sum_{v \in V} d(v) = 2|E|.$$

The *size* of a graph G is $|E(G)|$, the number edges in G.

Definition

Regular / k-regular graph

Definition

Subgraph

Definition

Spanning subgraph

Definition

Induced subgraph

Definition

Independent set / independence number

Definition

Complete graph

Definition

Edge deletion

Definition

Vertex deletion

Definition

Clique / clique number

Definition

Walk / (u, v)-walk

A graph is *complete* if each vertex is adjacent to each other vertex. The complete graph of order n is denoted by K_n.

Let G be a graph. An *edge deletion* is the process of removing an edge e from G. This process results in a new graph, denoted by $G - e$, where

$$V(G-e) = V(G) \quad \text{and} \quad E(G-e) = E(G) - \{e\}.$$

Let G be a graph. A *vertex deletion* is the process of removing a vertex v from G to create a new graph, denoted by $G - v$, where

$$V(G-v) = V(G) - \{v\}$$
$$E(G-v) = \{e \in E(G) : v \notin e\}.$$

Let G be a graph. A subset of vertices $W \subseteq V(G)$ is called a *clique* if $G[W]$ is a complete graph. The *clique number* of G is the size of the largest clique in G.

Let G be a graph. A *walk* in G is a sequence of vertices $W = (v_0, v_1, v_2, \ldots, v_n)$ where each vertex adjacent to the next, that is, $v_0 \sim v_1 \sim v_2 \sim \cdots \sim v_n$. A (u, v)-*walk* is a walk whose first vertex is u and whose last vertex is v.

A graph $G = (V, E)$ is *regular* if all of the vertices in the graph has the same degree. Moreover, the graph is k-*regular* if $d(v) = k$ for all $v \in V$.

G is k-regular	$\Leftrightarrow$	$d(v) = k$ for all $v \in V(G)$

Let G and H be graphs. The graph H is a *subgraph* of G if $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$.

Let G and H be graphs. The graph H is a *spanning subgraph* of G if H is a subgraph of G and $V(H) = V(G)$.

Let G be a graph and let W be a subset of vertices is G (that is, $W \subseteq V(G)$). The *induced subgraph* of G onto W is the new graph, denoted by $G[W]$, where

$$V(G[W]) = W,$$
$$E(G[W]) = \{\{u, v\} \in E(G) : u \in W, v \in W\}.$$

Let G be a graph. A subset of vertices $W \subseteq V(G)$ is *independent* if no two vertices in W are adjacent. The *independence number* of G is the size of the largest independent set in G.

Definition

Walk length

Definition

Concatenation

Definition

Path / (u, v)-path

Definition

Connected vertices

Definition

Connected graph

Definition

Component

Definition

Cut vertex

Definition

Cut edge

Definition

Cycle

Definition

Tree

Let W_1 be a (u,v)-walk and W_2 be a (v,w)-walk. The *concatenation* of W_1 and W_2 (denoted by $W_1 + W_2$) is the (u,w)-walk defined by the walk W_1, followed by the walk W_2.

The *length* of a walk is the number of edges traversed by the walk.

Let G be a graph. The vertices $u, v \in V(G)$ are *connected* if G contains a (u,v)-path.

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u is connected to v	$\Leftrightarrow$	G contains a (u,v)-path

A *path* in a graph is a walk in which no vertex is repeated. A (u,v)-path is a path from u to v.

A *component* H of a graph G is a maximal connected subgraph of G, meaning that H is not a proper subgraph of any connected subgraph of G.

A graph G is *connected* if, for every pair of vertices $u, v \in V(G)$, u and v are connected.

G is connected	$\Leftrightarrow$	for every $u, v \in V(G)$, u is connected to v

Let G be a graph. An edge $e \in E(G)$ is a *cut edge* if $G - e$ has more components than G.

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v is a cut edge	$\Leftrightarrow$	$G - e$ has more components than G

Let G be a graph. A vertex $v \in V(G)$ is a *cut vertex* if $G - v$ has more components than G.

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v is a cut vertex	$\Leftrightarrow$	$G - v$ has more components than G

A connected graph G is a *tree* if it does not contain any cycles.

G is a tree	$\Leftrightarrow$	G does not contain any cycles

A *cycle* is a walk of length at least three whose first vertex and last vertex are the same.

Definition

Forest

Definition

Spanning tree

Definition

Hamiltonian cycle / Hamiltonian graph

Definition

Eulerian trail / Eulerian graph

Let G be a graph. A *spanning tree* of G is a spanning subgraph of G that is a tree.

A graph G is a *forest* if every component of G is a tree.

G is a forest	$\Leftrightarrow$	each component of G is a tree

Let G be a graph. An *Eulerian trail* is a walk in G that traverses every edge exactly once. A graph G is *Eulerian* if it contains an Eulerian trail.

Let G be a graph. A *Hamiltonian cycle* is a cycle of G that contains all of the vertices of G. A graph G is *Hamiltonian* if it contains a Hamiltonian cycle.