QUIZ 7
University of Massachusetts Amherst
Math 235 – Spring 2014
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NAME:

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(1) (2 points each) Compute the characteristic polynomial of the matrix $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$.

ANSWER:

$$\begin{aligned} \det(A - \lambda I) &= \det \begin{bmatrix} -\lambda & 1 & 0 \\ 0 & -\lambda & 1 \\ 1 & 0 & -\lambda \end{bmatrix} \\ &= -\lambda \begin{vmatrix} -\lambda & 1 \\ 0 & -\lambda \end{vmatrix} + 1 \begin{vmatrix} 1 & 0 \\ -\lambda & 1 \end{vmatrix} \\ &= -\lambda^3 + 1. \end{aligned}$$

(2) (2 points) Find an eigenvector of $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ corresponding to the eigenvalue $\lambda = 1$.

ANSWER:

$$\begin{aligned} \ker(A - I) &= \ker \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 1 & 0 & -1 \end{bmatrix} \\ &= \ker \begin{bmatrix} 1 & 0 & -1 \\ 0 & -1 & 1 \\ -1 & 1 & 0 \end{bmatrix} \quad \text{(swap first and last row)} \\ &= \ker \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 1 & -1 \end{bmatrix} \quad \text{(add first to third, multiply second by } -1\text{)} \\ &= \ker \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}. \end{aligned}$$

So $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

(3) (1 point each) True/False: circle **T** or **F**.

T **F** : The algebraic multiplicity of an eigenvalue is greater than or equal to its geometric multiplicity.

ANSWER: **True.** See Theorem 5 in the 7.3 notes.

T **F** : $\begin{bmatrix} 5 \\ 1 \end{bmatrix}$ is an eigenvector of $A = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$.

ANSWER: **True.**

$$A\vec{v} = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 5 \\ 1 \end{bmatrix} = \begin{bmatrix} 20 \\ 4 \end{bmatrix} = 4 \begin{bmatrix} 5 \\ 1 \end{bmatrix} = 4\vec{v}.$$

T **F** : 0 is an eigenvalue of $\begin{bmatrix} 10 & 15 \\ 4 & 6 \end{bmatrix}$.

ANSWER: **True.** The determinant of a matrix is equal to the product of its eigenvalues, and the determinant of this matrix is 0. Hence 0 must be an eigenvalue.

T **F** : If $\vec{v}$ is an eigenvector of A for the eigenvalue λ, then $2\vec{v}$ is an eigenvector of A for the eigenvalue λ.

ANSWER: **True.**

$$\begin{aligned} A(2\vec{v}) &= 2A\vec{v} = 2\lambda\vec{v} \quad (\text{since } A\vec{v} = \lambda\vec{v}) \\ &= \lambda(2v). \end{aligned}$$

T **F** : If $\vec{v}$ is an eigenvector of A for the eigenvalue λ, then $\vec{v}$ is an eigenvector of A^2 for the eigenvalue λ^2.

ANSWER: **True.**

$$\begin{aligned} A^2\vec{v} &= A(A\vec{v}) = A(\lambda\vec{v}) \quad (\text{since } A\vec{v} = \lambda\vec{v}) \\ &= \lambda A\vec{v} = \lambda\lambda\vec{v} = \lambda^2\vec{v}. \end{aligned}$$

T **F** : If $\vec{v}_1$ and $\vec{v}_2$ are eigenvectors of A where $A\vec{v}_1 = \vec{v}_1$ and $A\vec{v}_2 = 3\vec{v}_2$, then $\vec{v}_1 + \vec{v}_2$ is an eigenvector of A.

ANSWER: **False.**

$$A(\vec{v}_1 + \vec{v}_2) = A\vec{v}_1 + A\vec{v}_2 = \vec{v}_1 + 3\vec{v}_2$$

which is not a scalar multiple of $(\vec{v}_1+\vec{v}_2)$. That is, $A(\vec{v}_1+\vec{v}_2) \neq \lambda(\vec{v}_1+\vec{v}_2)$ for any λ.