QUIZ 8
University of Massachusetts Amherst
Math 235 – Spring 2014
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NAME:

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(1) (1 point) The matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 3 \end{bmatrix}$ is diagonalizable, meaning that $S^{-1}AS = D$ for some invertible matrix S and diagonal matrix D. Give one possibility for D.

ANSWER: D is a diagonal matrix with the eigenvalues of A on its diagonal. A is triangular, so the eigenvalues of A are 1, 2, and 3 (the values on its diagonal). One possibility for D is $D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

(2) (0.5 points each) Determine if the matrix is diagonalizable. Circle **Y** (yes) or **N** (no).

Y **N** : $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

ANSWER: Yes. This is already a diagonal matrix. Also, this matrix has one eigenvalue, 0, and the dimension of E_0 is 2.

Y **N** : $\begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$

ANSWER: Yes. This is already a diagonal matrix. Also, this matrix has distinct eigenvalues.

Y **N** : $\begin{bmatrix} 0 & 3 \\ 0 & 0 \end{bmatrix}$

ANSWER: No. This matrix has one eigenvalue, 0, and $\dim E_0 = 1$.

Y **N** : $\begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix}$

ANSWER: No. This matrix has one eigenvalue, 2, and $\dim E_2 = 1$.

Y **N** : $\begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$

ANSWER: Yes. This matrix has two distinct eigenvalues, 2 and -2.

Y **N** : $\begin{bmatrix} 0 & 2 \\ 2 & 3 \end{bmatrix}$

ANSWER: Yes. This matrix has two distinct eigenvalues, -1 and 4.

(3) (1 point each) True/False: circle **T** or **F**.

T **F** : A square matrix A is diagonalizable if and only if it is similar to a diagonal matrix.

ANSWER: True. This is the definition of *diagonalizable*.

T **F** : Every diagonal matrix is diagonalizable.

ANSWER: True. If D is a diagonal matrix, then D is diagonalizable because it is similar to itself.

T **F** : Every diagonal matrix is invertible.

ANSWER: False. For example: $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

T **F** : Every invertible matrix is diagonalizable

ANSWER: False. For example: $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$.

T **F** : If an $n \times n$ matrix has n distinct eigenvalues, then it is diagonalizable.

ANSWER: True. See notes.

T **F** : If an $n \times n$ matrix is diagonalizable, then it has n distinct eigenvalues.

ANSWER: False. For example: $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.