1.1 — Introduction to linear systems
University of Massachusetts Amherst
Math 235 — Spring 2014

Linear algebra finds its origins in the study of *linear systems of equations*. Some familiar examples of *linear equations* are

$$ax + by = c \qquad \text{and} \qquad ax + by + cz = d.$$

There is no limit to the number of variables a linear equation can take; a linear equation in n variables has the form

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = b,$$

where $x_1, \ldots, x_n$ are variables and $a_1, \ldots, a_n, b$ are constants, often referred to as coefficients.

Example 1. Solve the system of equations $\left\{ \begin{array}{r} x - 2y = 1 \\ 2x + 3y = 1 \end{array} \right\}$.

The system of equations in this example has a unique solution.

How many solutions do the systems $\left\{ \begin{array}{r} y - x = 1 \\ y - x = 0 \end{array} \right\}$ and $\left\{ \begin{array}{r} y - x = 1 \\ 2y - 2x = 2 \end{array} \right\}$ have?

Example 2. Find all solutions to the system of equations $\left\{\begin{array}{r} x+2y+3z=0 \\ 4x+5y+6z=3 \\ 7x+8y+9z=0 \end{array}\right\}$.

Definition 3. A system of linear equations is *inconsistent* if it has no solutions. Otherwise, the system is *consistent*.

Questions: How many solutions can a system of linear equations have? Are there more efficient methods for determining whether or not a system is consistent?

ADDITIONAL EXERCISES

(1) Find all solutions of the linear systems. Give a geometric description of your solutions in terms of intersecting planes.

(a) $\left\{\begin{array}{r} x+4y+z=0 \\ 4x+13y+7z=0 \\ 7x+22y+13z=1 \end{array}\right\}$ (b) $\left\{\begin{array}{r} x+y-z=0 \\ 4x-y+5z=0 \\ 6x+y+4z=0 \end{array}\right\}$ (c) $\left\{\begin{array}{r} x+4y+z=0 \\ 4x+13y+7z=0 \\ 7x+22y+13z=0 \end{array}\right\}$

(2) Consider the system $\left\{\begin{array}{r} x+y-z=-2 \\ 3x-5y+13z=18 \\ x-2y+5z=k \end{array}\right\}$ where k is an arbitrary number.

(a) For which value(s) of k is this system consistent?
(b) For each value of k you found in part (a), how many solutions does the system have?
(c) Find all solutions for each value of k.

(3) Find all polynomials $f(t)$ of degree ≤ 2 (i.e. $f(t)=at^2+bt+c$) that satisfy $f(1)=1$, $f(2)=0$, and $\int_1^2 f(t)\,dt=-1$.