2.2 — Linear transformations in space
University of Massachusetts Amherst
Math 235 — Spring 2014

Let A be a matrix representing a linear transformation of the plane $T\colon \mathbb{R}^2 \to \mathbb{R}^2$.

How does the matrix $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ act on vectors $\begin{bmatrix} x \\ y \end{bmatrix}$ in the plane?

How does the matrix $A = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$ act on vectors $\begin{bmatrix} x \\ y \end{bmatrix}$ in the plane?

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The matrix $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ gives a rotation by the angle θ about the origin. How do the matrices $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ and $\frac{1}{2}\begin{bmatrix} \sqrt{2} & -\sqrt{2} \\ \sqrt{2} & \sqrt{2} \end{bmatrix}$ act on the plane?

Example 1. Recall that $\text{proj}_{\vec{v}}(\vec{x})$ is the orthogonal projection of the vector $\vec{x}$ onto $\vec{v}$, which is defined by

$$\text{proj}_{\vec{v}}(\vec{x}) = \frac{\vec{v} \cdot \vec{x}}{\vec{v} \cdot \vec{v}} \vec{v}.$$

The projection map is a linear transformation.

(i) Find the matrix corresponding to $\text{proj}_{\vec{v}}$ if $\vec{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

(ii) Compute $\text{proj}_{\vec{v}}(\vec{x})$ for $\vec{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\vec{x} = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$.

ADDITIONAL EXERCISES

(1) Give a geometric interpretation for the linear transformation $T(\vec{x}) = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \vec{x}$.

(2) Give a geometric interpretation for the linear transformation $T(\vec{x}) = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \vec{x}$.

(3) Give a geometric interpretation for the linear transformation $T(\vec{x}) = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \vec{x}$.

(4) Let T be a linear transformation given by $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and S be a linear transformation given by $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$. Is it true that $S(T(\vec{x})) = T(S(\vec{x}))$? Show or Explain.