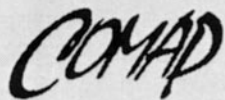


UMAP

Modules in
Undergraduate
Mathematics
and Its
Applications

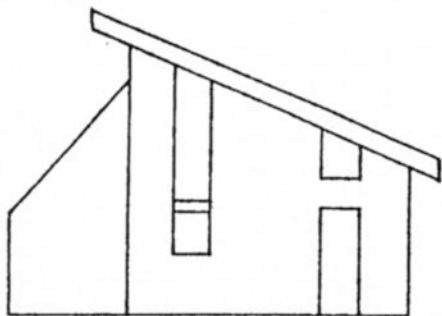
Published in
cooperation with
the Society
for Industrial
and Applied
Mathematics, the
Mathematical
Association of
America, the
National Council
of Teachers of
Mathematics,
the American
Mathematical
Association of
Two-Year Colleges,
The Institute
of Management
Sciences, and the
American Statistical
Association.

The logo for COMAP, consisting of the letters 'COMAP' in a stylized, cursive, handwritten font.

Module 558

Passive Solar Heating

Simon Cohen
Jay Kappraff



Applications of Linear Algebra to
Architecture and Engineering

MODULES AND MONOGRAPHS IN UNDERGRADUATE
MATHEMATICS AND ITS APPLICATIONS (UMAP) PROJECT

The goal of UMAP was to develop, through a community of users and developers, a system of instructional modules in undergraduate mathematics and its applications to be used to supplement existing courses and from which complete courses may eventually be built.

The Project was guided by a National Advisory Board of mathematicians, scientists, and educators. UMAP was funded by a grant from the National Science Foundation and is now supported by the Consortium for Mathematics and Its Applications (COMAP), Inc., a non-profit corporation engaged in research and development in mathematics education.

COMAP STAFF

| | |
|----------------------|---------------------------|
| Paul J. Campbell | Editor |
| Solomon A. Garfunkel | Executive Director, COMAP |
| Laurie Aragón | Development Director |
| Roland D. Cheyney | Project Manager |
| Philip A. McGaw | Production Manager |
| Laurie M. Holbrook | Copy Editor |
| Dale Horn | Design Assistant |
| Robin Altomonte | Distribution |

The Project would like to thank Charles Diminine of St. Bonaventure University, J.W. Goldston of Pembroke State University, Raymond Greenwell of Albion College, and one anonymous reviewer for their reviews, and all others who assisted in the production of this unit.

This material was prepared with the support of National Science Foundation Grant No. SED80-07731. Recommendations expressed are those of the author and do not necessarily reflect the views of the NSF or of COMAP.

PASSIVE SOLAR HEATING

by

Simon Cohen

and

Jay Kappraff

Department of Mathematics

New Jersey Institute of Technology

Newark, New Jersey 07102

TABLE OF CONTENTS

| | | |
|----|---|----|
| 1. | INTRODUCTION | 1 |
| 2. | BASIC PRINCIPLES | 1 |
| 3. | HEAT CONDUCTION THROUGH A WALL | 4 |
| | 3.1 Formulating the Problem | 4 |
| | 3.2 Obtaining the Equations | 5 |
| | 3.3 Diagraming the Equations | 7 |
| | 3.4 Representing the Equations in Matrix Form | 9 |
| | 3.5 Solving the Equations I | 11 |
| | 3.6 Example I | 15 |
| 4. | TEMPERATURE DISTRIBUTION IN A HOUSE | 20 |
| | 4.1 Preliminary Remarks | 20 |
| | 4.2 The House Specifications | 21 |
| | 4.3 Graphical Representation of the House | 21 |
| | 4.4 The House Temperature Equations | 24 |
| | 4.5 Solving the Equations II | 26 |
| 5. | ANSWERS TO EXERCISES | 41 |
| | SPECIAL ASSISTANCE SUPPLEMENT | 44 |
| | APPENDIX | 45 |

Intermodular Description Sheet: UMAP Unit 558

Title: PASSIVE SOLAR HEATING

Authors: Simon Cohen
and
Jay Kappraff
Department of Mathematics
New Jersey Institute of Technology
Neward, NJ 07102

Review Stage/Date: III 5/7/82

Mathematics Field: Linear algebra

Applications Field: Architecture, engineering

Target Audience: Students in architecture, engineering, or linear algebra courses.

Abstract: A problem in solar heating is formulated and solved in a mathematical context. The primary aim of this unit is to acquaint the student with some of the underlying principles and practices of solar heating, to illustrate mathematical modeling, and to introduce concepts of numerical linear algebra.

Prerequisites: The student should be taking, or have taken, a basic course in linear algebra.

1. INTRODUCTION

The sun provides us with many sources of free energy. There are many different ways in which this energy can be harvested. For example, we can capture the energy of mountain streams whose flow is the result of the evaporation of water by the sun and its subsequent reappearance in the form of rainfall. We can capture the winds which come about due to atmospheric temperature variations induced by the sun. The sun's rays can be turned into electricity through the use of photosensitive cells. There is one other way to harness the sun's energy that is of great importance to architects. Buildings can be designed to efficiently capture the energy that ordinarily enters a building through its windows, walls, and roof. These passive solar gains provide people with the opportunity to dramatically reduce their fuel bills.

If more than half of the total energy requirements for a building are provided by passive solar means, then we say that the building is a passive-solar heated structure.

Passive solar heating works very well. This has been demonstrated time and again in many different kinds of buildings located in a wide variety of climates. The occupants of these buildings testify to their comfort, the ease of operation of passive solar systems, and especially to their low fuel bills. A big problem, however, has been the lack of a quantitative basis of incorporating the basic concepts of passive solar heating into architectural design. In this unit we will try to demonstrate how passive solar heating works and how to calculate its effectiveness.

2. BASIC PRINCIPLES

Terms such as "temperature," "thermal energy" (or "heat energy," or simply "heat") have precise, and rather complex, physical definitions. We shall treat them in a manner similar to the way the terms "point" and "line" are treated in elementary geometry. That is, we shall not define them in terms of other physical quantities, but shall rely on your intuitive understanding of these terms and simply state some basic principles involved in their use.

If you stir a bowl of hot soup, what happens to the handle of the spoon? Why? Does the heat energy flow from the cooler to the warmer part of the spoon or from the warmer to the cooler part? Does the heat energy flow faster when the difference between the room temperature and the

soup temperature is 100° or when it is 1° ? Answer the questions in this paragraph before reading further.

You have, in answering the above questions, discovered that you already know the main laws for heat conduction in a qualitative form. The handle of the spoon gets hot because the heat energy flows from the warmer to the cooler parts of the spoon. The greater the temperature difference, the faster the heat energy flows.

We may summarize the laws of heat conduction as follows:

- I. Heat flow is in the direction of decreasing temperature.
- II. Rate of heat flow is proportional to temperature difference.
- III. The quantity of heat gained or lost by a substance when its temperature changes is proportional to the weight of the substance and to the temperature change.

In the English system of units heat energy is measured in British Thermal Units (BTU). One BTU is the amount of heat energy required to increase the temperature of a pound of water one degree Fahrenheit.

We may write Principle III in the form of an equation as

$$(1) \quad Q = CW(\Delta T)$$

where

Q is the amount of heat gained or lost by the substance (measured in BTU's),

C is the constant of proportionality, dependent on the substance, called the specific heat of the substance,

W is the weight of the substance (measured in pounds),

ΔT is the temperature change undergone by the substance (measured in degrees Fahrenheit).

Exercise 1. What is the specific heat of water?

If ρ is the density of the substance, then $W = \rho V$, where V is the volume of the substance, and we may rewrite Equation (1) as

$$(2) \quad Q = C\rho V(\Delta t)$$

Letting $S = C\rho$ we get

$$(3) \quad Q = SV(\Delta T)$$

or

$$(4) \quad S = \frac{Q}{V(\Delta T)}$$

The constant S is known as the volumetric specific heat of the substance.

A rearrangement of Equation (4) produces

$$(5) \quad \Delta T = \frac{Q}{SV}$$

Thus a substance with a large S value acts like a "heat sponge" and absorbs a lot of heat for only a small change in temperature. On the other hand, if S is small, then only a small amount of heat absorption will result in a large temperature change. In designing passive solar heated buildings we will therefore want to use materials with large S values in order to store heat energy at low temperatures. (See Table 1.)

TABLE I

Physical Properties of Some Building Materials

| MATERIAL | ρ (lb./cu. ft.) | C (BTU/ lb. ^o F) | S (BTU/ cu. ft. ^o F) | K* BTU/ hr. ft. ^o F) |
|-----------|----------------------------|-----------------------------------|---------------------------------------|---------------------------------------|
| BRICK | 144 | 0.20 | 28.8 | 0.58 |
| MASONRY | 106 | 0.20 | 21.2 | 0.38 |
| CONCRETE | 144 | 0.20 | 28.8 | 0.54 |
| LIMESTONE | 105 | 0.22 | 23.1 | 0.40 |
| GLASS | 170 | 0.2 | 34.0 | 0.45 |
| OAK WOOD | 51 | 0.57 | 29.07 | 0.16 |
| PINE WOOD | 51 | 0.57 | 29.07 | 0.1 |
| CORK | 10 | 0.04 | 0.4 | 0.025 |
| RUBBER | 75 | 0.48 | 36 | 0.087 |

*See Equation 9, Section 3.2.

3. HEAT CONDUCTION THROUGH A WALL

3.1 Formulating the Problem

Let us first consider a wall made of a uniform material, one side of which is exposed to the outdoors, the other side being indoors. The temperature in the wall will, of course, vary since the temperature must adjust itself through the wall from outdoor to indoor values. Finding the exact temperature distribution is a very difficult problem; however if the wall is high enough and wide enough, we can assume that most of the change occurs across the thickness only (see Exercise 2). Assume the wall has thickness L with outdoor temperature T_a and indoor temperature T_b . We denote the temperature at time t and position x along the thickness of the wall by the function $T(x,t)$. Finally, we imagine the wall to be divided into n sections by placing n evenly spaced points x_1, x_2, \dots, x_n across its thickness. (See Figure 1. n is an arbitrary positive integer.)

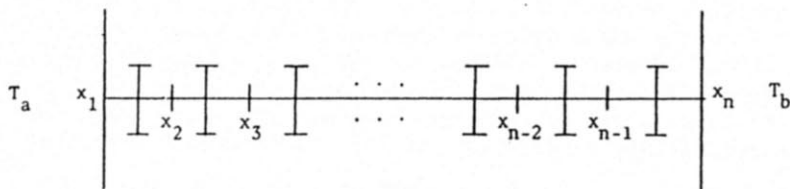


Figure 1. Partitioning the wall.

Each of the interior points x_2, x_3, \dots, x_{n-1} is the midpoint of a section of length $\Delta x = L/(n-1)$, while the endpoints x_1 and x_n are included in intervals of length $\Delta x/2$. We simplify the computations by assuming that all the mass of a section is accumulated at the point enclosed by that section. Thus the wall is, in effect, replaced by n points x_1, \dots, x_n , referred to as nodes. In the following, we will refer to the section enclosing node x_i as the i th section.

Our objective is to determine the temperature across the wall at any time in response to the changing temperature conditions both outdoors and indoors. To carry out this analysis, we make an assumption about the flow of time analogous to the assumption we made about the accumulation of mass at points x_1, x_2, \dots, x_n .

