This homework covers sections 2.2–2.4. It is due in class Wednesday, February 28. Hand in a hardcopy of your solutions.

Recall that the `>>>` operator shifts a 0 into the leftmost position.

Write your answers carefully, giving attention to clearly presenting the solution, and be sure to always show your work and explain your reasoning. Unsupported answers will receive little or no credit.

While you may discuss problems with other students, you should always make the first attempt on a problem yourself and **you must write up your own solutions in your own words**. *You may not collaboratively write solutions or copy a solution that one person in the group writes up.*

1. Let A, B, and C be sets. Simplify each of the following expressions; as part of this, the complement operator should only be applied to individual sets. Justify each step in the simplification with one of the rules of set theory.

   a) $A \cup (B \cup A)$ b) $(A \cap B) \cap \overline{A}$ c) $(A \cup B) \cap \overline{B}$

   d) $(A \cup B) \cup (A \cap B)$ e) $\overline{\overline{A} \cup \overline{B} \cap C}$ f) $\overline{\overline{A} \cap \overline{\overline{B} \cap \overline{C}}}$

   g) $\overline{A \cap \overline{\overline{A} \cup B}}$

2. Let a, b, and c be values of type `int` given as hexadecimal numbers in Java as

   ```
   a = 0xABCD1234      b = 0x5678EF00      c = 0xFFFF
   ```

   Find the values of the following Java expressions, writing the answers as hexadecimal numbers. Show your work to show how the answer is computed; do not simply give the answer.

   a) `(a << 16) | (b >>> 16)`   b) `(a & (c << 16)`

   c) `(a & (c << 16)) | (b & c)`   d) `(a & b) | ~(c << 8)`

3. Let m and n be the 16-bit binary numbers shown below.

   $$n = 1011\ 0010\ 1010\ 0011$$
   $$m = 0001\ 1000\ 1110\ 0110$$

   (a) Write the value of each of the following as a 16-bit binary number.

        (i) n & m (ii) $n \mid m$ (iii) $\sim m$

   (b) Write out the subset of $\{15, 14, 13, \ldots, 2, 1, 0\}$ that each of the following corresponds to. For example, `1100 0000 0000 0001` corresponds to the set $\{15, 14, 0\}$.

        (i) m (ii) n (iii) n & m (iv) $n \mid m$ (v) $\sim m$

4. What is the following function counting? Give a concise English statement, and explain your answer. (Hint: write n in binary, and trace through the code using a specific example.)

```
int countSomething ( int n ) {
  int count = 0;
  for ( int i = 0 ; i < 32 ; i++ ) {
    if ( ( n & 1 ) == 1 ) {
      count++;
    }
    n = n >>> 1;
  }
  return count;
}
```

5. Describe the set $\{1, 2, 3\} \times \mathbb{Z}$. Show that you understand its structure.

6. Consider the following functions from $\mathbb{N}$ to $\mathbb{N}$. For each function, determine whether the function is *one-to-one* and whether the function is *onto*. Justify your answers!

a) $f(n) = n^2 + n + 1$

b) $f(n) = \begin{cases} n+1 & \text{if } n \text{ is even} \\ n-1 & \text{if } n \text{ is odd} \end{cases}$

c) $f(n) = \begin{cases} n/2 & \text{if } n \text{ is even} \\ (n+1)/2 & \text{if } n \text{ is odd} \end{cases}$