Proof by Induction

We can prove a statement *P(x)* for any natural number *x* by showing a chain of statements

P(0), P(0) → P(1), P(1) → P(2), P(2) → P(3), …

Thus, prove

P(0) ∧ ∀k (P(k) → P(k+1))

- *proof by induction*
  - base case: show P(0)
  - inductive case: show ∀k (P(k) → P(k+1))
    - tactic: let *k* be an arbitrary element of ℕ and prove *P(k)* → *P(k+1)*
      - P(k) is the *inductive hypothesis*
    - another tactic: show (P(0) ∧ P(1) ∧ P(2) ∧ … ∧ P(k)) → P(k+1)
      - assume *P(x)* holds for all *x* 0 to *k*, then show P(k+1)

1. Use induction to prove that $n^3 + 3n^2 + 2n$ is divisible by 3 for all natural numbers n.

2. Use induction to prove that

$$\sum_{i=0}^{n} r^i = \frac{1 - r^{n+1}}{1 - r}$$

for any natural number n and for any real number r such that $r \neq 1$.

3. Use induction to prove that for any natural number n,

$$\sum_{i=0}^{n} \frac{1}{2^i} = 2 - \frac{1}{2^n}$$

4. Use induction to prove that for any natural number n,

$$\sum_{i=0}^{n} 2^i = 2^{n+1} - 1$$

5. Use induction to prove that for any positive integer n,

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

6. Use induction to prove that for any positive integer n,

$$\sum_{i=1}^{n} (2i - 1) = n^2$$

10. Use induction to prove the following generalized distributive laws for propositional logic: For any natural number $n > 1$ and any propositions q, p_1, p_2, $\ldots$, p_n,

a) $q \wedge (p_1 \vee p_2 \vee \cdots \vee p_n) = (q \wedge p_1) \vee (q \wedge p_2) \vee \cdots \vee (q \wedge p_n)$

b) $q \vee (p_1 \wedge p_2 \wedge \cdots \wedge p_n) = (q \vee p_1) \wedge (q \vee p_2) \wedge \cdots \wedge (q \vee p_n)$