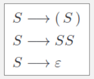
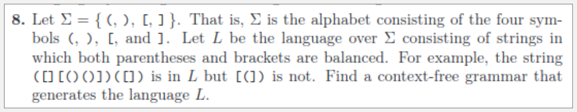
Applications

- balancing parens



$$S \longrightarrow (S)$$
$$S \longrightarrow SS$$
$$S \longrightarrow \varepsilon$$



8. Let $\Sigma = \{$ (,), [,] $\}$. That is, Σ is the alphabet consisting of the four symbols (,), [, and]. Let L be the language over Σ consisting of strings in which both parentheses and brackets are balanced. For example, the string ([][()()])([]) is in L but [(]) is not. Find a context-free grammar that generates the language L.

Matching and balancing are things that context-free grammars can express but regular expressions cannot.

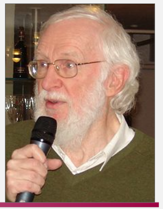
Applications

- aspects of real languages (natural languages, programming languages) can be expressed with context-free grammars
  - provides a precise definition of legal syntax
  - provides an algorithm for parsing

- *Backus-Naur Form* (BNF) is a notation typically used in these applications
  - there are variations

Backus and Naur

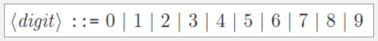
- John Backus, 1924-2007
  - American computer scientist
  - also known for Fortran (1950s)
    - first widely-used high-level programming language
  - received the 1977 Turing Award for “profound, influential, and lasting contributions to the design of practical high-level programming systems”



- Peter Naur, 1928-2016
  - Danish computer scientist
  - also known for ALGOL 60 (1960)
    - introduced many influential features (block structure, nested functions, lexical scope)
  - received the 2005 Turing Award for work on ALGOL 60

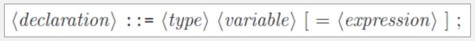
BNF

- non-terminals typically have meaningful names rather than being single symbols
  - written ⟨*thing*⟩ to distinguish from terminals
- terminals are the elements of the language
  - also typically multi-symbol units
- uses `::=` instead of →
- offers a more compact representation for related rules



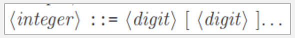
$$\langle \mathit{digit} \rangle \ ::= 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9$$

| denotes alternatives



$$\langle \mathit{declaration} \rangle \ ::= \langle \mathit{type} \rangle \ \langle \mathit{variable} \rangle \ [= \langle \mathit{expression} \rangle \ ] \ ;$$

[ ] denotes optional



$$\langle \mathit{integer} \rangle \ ::= \langle \mathit{digit} \rangle \ [\ \langle \mathit{digit} \rangle \ ] \ldots$$

[ ] . . . denotes 0 or more repetitions

  - parens used for grouping

⟨*sentence*⟩ ::= ⟨*simple-sentence*⟩ [and ⟨*simple-sentence*⟩]...
⟨*simple-sentence*⟩ ::= ⟨*nout-part*⟩ ⟨*verb-part*⟩
⟨*noun-part*⟩ ::= ⟨*article*⟩ ⟨*noun*⟩ [who ⟨*verb-part*⟩]...
⟨*verb-part*⟩ ::= ⟨*intransitive-verb*⟩ | (⟨*transitive-verb*⟩ ⟨*noun-part*⟩)
⟨*article*⟩ ::= the | a
⟨*noun*⟩ ::= man | woman | dog | cat | computer
⟨*intransitive-verb*⟩ ::= runs | jumps | hides
⟨*transitive-verb*⟩ ::= knows | loves | chases | owns

⟨*sentence*⟩ ⟹ ⟨*simple-sentence*⟩
⟹ ⟨*noun-part*⟩ ⟨*verb-part*⟩
⟹ ⟨*article*⟩ ⟨*noun*⟩ ⟨*verb-part*⟩
⟹ the ⟨*noun*⟩ ⟨*verb-part*⟩
⟹ the man ⟨*verb-part*⟩
⟹ the man ⟨*transitive-verb*⟩ ⟨*noun-part*⟩
⟹ the man loves ⟨*noun-part*⟩
⟹ the man loves ⟨*article*⟩ ⟨*noun*⟩ who ⟨*verb-part*⟩
⟹ the man loves a ⟨*noun*⟩ who ⟨*verb-part*⟩
⟹ the man loves a woman who ⟨*verb-part*⟩
⟹ the man loves a woman who ⟨*intransitive-verb*⟩
⟹ the man loves a woman who runs

⟨*statement*⟩ ::= ⟨*block-statement*⟩ | ⟨*if-statement*⟩ | ⟨*while-statement*⟩
| ⟨*assignment-statement*⟩ | ⟨*null-statement*⟩
⟨*block-statement*⟩ ::= { [⟨*statement*⟩]... }
⟨*if-statement*⟩ ::= if "(" ⟨*condition*⟩ ")" ⟨*statement*⟩ [else ⟨*statement*⟩]

⟨*while-statement*⟩ ::= while "(" ⟨*condition*⟩ ")" ⟨*statement*⟩
⟨*assignment-statement*⟩ ::= ⟨*variable*⟩ = ⟨*expression*⟩ ;
⟨*null-statement*⟩ ::= ε

⟨*expression*⟩ ::= ⟨*term*⟩ [[+ | −] ⟨*term*⟩]...
⟨*term*⟩ ::= ⟨*factor*⟩ [[∗ | /] ⟨*factor*⟩]...
⟨*factor*⟩ ::= ***ident*** | **number** | "(" ⟨*expression*⟩ ")"

- quotes ("") are being used here to distinguish terminals [,], (,) in the language from the BNF notation [,], (,)
- **`ident`** refers to an identifier, **`number`** refers to a number

2. Rewrite the example BNF grammar for a subset of English as a context-free grammar.

⟨*sentence*⟩ ::= ⟨*simple-sentence*⟩ [and ⟨*simple-sentence*⟩]...
⟨*simple-sentence*⟩ ::= ⟨*nout-part*⟩ ⟨*verb-part*⟩
⟨*noun-part*⟩ ::= ⟨*article*⟩ ⟨*noun*⟩ [who ⟨*verb-part*⟩]...
⟨*verb-part*⟩ ::= ⟨*intransitive-verb*⟩ | (⟨*transitive-verb*⟩ ⟨*noun-part*⟩)
⟨*article*⟩ ::= the | a
⟨*noun*⟩ ::= man | woman | dog | cat | computer
⟨*intransitive-verb*⟩ ::= runs | jumps | hides
⟨*transitive-verb*⟩ ::= knows | loves | chases | owns

3. Write a single BNF production rule that is equivalent to the following context-free grammar:

$$\begin{aligned} S &\longrightarrow aSa \\ S &\longrightarrow bB \\ B &\longrightarrow bB \\ B &\longrightarrow \varepsilon \end{aligned}$$