

1. Find a derivation for the string  $caabcb$ , according to the first example grammar in this section. Find a derivation for the string  $aabbcc$ , according to the second example grammar in this section. Find a derivation for the string  $aaaa$ , according to the third example grammar in this section.

$S \rightarrow SABC$	$S \rightarrow SABC$	$S \rightarrow DTE$
$S \rightarrow \varepsilon$	$S \rightarrow X$	$T \rightarrow BTA$
$AB \rightarrow BA$	$BA \rightarrow AB$	$T \rightarrow \varepsilon$
$BA \rightarrow AB$	$CA \rightarrow AC$	$BA \rightarrow AaB$
$AC \rightarrow CA$	$CB \rightarrow BC$	$Ba \rightarrow aB$
$CA \rightarrow AC$	$XA \rightarrow aX$	$BE \rightarrow E$
$BC \rightarrow CB$	$X \rightarrow Y$	$DA \rightarrow D$
$CB \rightarrow BC$	$YB \rightarrow bY$	$Da \rightarrow aD$
$A \rightarrow a$	$Y \rightarrow Z$	$DE \rightarrow \varepsilon$
$B \rightarrow b$	$ZC \rightarrow cZ$	
$C \rightarrow c$	$Z \rightarrow \varepsilon$	

$$n_a(w) = n_b(w) = n_c(w)$$

$$a^n b^n c^n$$

$$a^{n^2}$$

3. Find a grammar that generates the language  $L = \{w \in \{a, b, c, d\}^* \mid n_a(w) = n_b(w) = n_c(w) = n_d(w)\}$ . Let  $\Sigma$  be any alphabet. Argue that the language  $\{w \in \Sigma^* \mid \text{all symbols in } \Sigma \text{ occur equally often in } w\}$  can be generated by a grammar.

4. For each of the following languages, find a grammar that generates the language. In each case, explain how your grammar works.

a)  $\{a^n b^n c^n d^n \mid n \in \mathbb{N}\}$

b)  $\{a^n b^m c^{nm} \mid n \in \mathbb{N} \text{ and } m \in \mathbb{N}\}$

c)  $\{ww \mid w \in \{a, b\}^*\}$

d)  $\{www \mid w \in \{a, b\}^*\}$

e)  $\{a^{2^n} \mid n \in \mathbb{N}\}$

f)  $\{w \in \{a, b, c\}^* \mid n_a(w) > n_b(w) > n_c(w)\}$

## Languages as Computation

- a *recursively enumerable language* is one for which there is a program whose output is exactly the strings in the language
- the languages generated by grammars are recursively enumerable languages
- a grammar is a model of computation with power equivalent to a computer program