Show the following logical equivalences by finding a chain of equivalences from the left side to the right. State which definition or law of logic justifies each equivalent in the chain.

(a) $p \wedge (q \wedge p) \equiv p \wedge q$

(b) $(\neg p) \rightarrow q \equiv p \vee q$

(c) $(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r$

Answer:

(a)
$$\begin{aligned} p \wedge (q \wedge p) &\equiv p \wedge (p \wedge q) && \text{Commutative Law} \\ &\equiv (p \wedge p) \wedge q && \text{Associative Law} \\ &\equiv p \wedge q && \text{Idempotent Law} \end{aligned}$$

(b)
$$\begin{aligned} (\neg p) \rightarrow q &\equiv \neg(\neg p) \vee q && \text{definition of } \rightarrow \\ &\equiv p \vee q && \text{Double Negation Law} \end{aligned}$$

(c)
$$\begin{aligned} (p \rightarrow r) \wedge (q \rightarrow r) &\equiv (\neg p \vee r) \wedge (q \rightarrow r) && \text{definition of } \rightarrow \\ &\equiv (\neg p \vee r) \wedge (\neg q \vee r) && \text{definition of } \rightarrow \\ &\equiv (r \vee \neg p) \wedge (\neg q \vee r) && \text{Commutative Law} \\ &\equiv (r \vee \neg p) \wedge (r \vee \neg q) && \text{Commutative Law} \\ &\equiv r \vee (\neg p \wedge \neg q) && \text{Distributive Law} \\ &\equiv r \vee \neg(p \vee q) && \text{DeMorgan's Law} \\ &\equiv \neg(p \vee q) \vee r && \text{Commutative Law} \\ &\equiv (p \vee q) \rightarrow r && \text{definition of } \rightarrow \end{aligned}$$