

The same comments from homework 3 about what constitutes a well-written proof apply here too.

Be careful to indicate when you are writing something that you wish to show and something that is assumed, both to distinguish these cases from each other and to distinguish them from deductions.

A common math error:  $3(3^k) - 1 \neq 3(3^k - 1)$

Connect the dots from the end of your deduction to what it is that you are showing. In #3,  $3^0 - 1 = 0$  is not quite enough to show that  $3^n - 1$  is divisible by 2 for  $n = 0$  — also note that 0 is divisible by 2. For #4, the correct answer for the recursive `sum` is the sum of the first  $n$  numbers, and for the loop invariant, the correct answer is that  $s$  is the sum of `arr[0..i-1]`. Simply stating that the return value (or the value of  $s$ ) is `arr[0]` isn't quite enough — explain that `arr[0]` is the sum of the first  $n$  numbers for  $n = 1$ , and that `arr[0]` is the sum of `arr[0..i-1]` for  $i = 1$ .

For the induction step with loop invariants, the strategy is to assume that the invariant is true at the beginning at some iteration, then show that what the code does over the next loop iteration means that the invariant is true at the beginning of the next iteration. In #4b, assume for  $i = k$ :  $s$  is the sum of elements `arr[0..k-1]`. The loop body adds `arr[k]` to  $s$  and increments  $i$  to  $k + 1$ , so at the next iteration, when  $i = k + 1$ ,  $s$  is the sum of elements `arr[0..k]` as required.

`...` is an informal notation, so include a minimum of two terms before employing `...` to define a set. (`...` indicates “and so on in the same manner”, not necessarily  $+1$ .) For example,  $\{0, 1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16, 17, 19, \dots\}$  — what's the next term? 20? 21?

For #8, again connect the dots: it was common to observe that  $x \in (A \cap B) \rightarrow x \in A \wedge x \in B$  (an element in the intersection is in each of the sets) and then conclude that  $x \in B$  means  $x \in A$  and thus  $B \subseteq A$ . The missing piece is to use the definition of  $=$ .  $B = A \cap B$  means  $x \in B \rightarrow x \in (A \cap B)$  (as well as the other direction) so then you have the necessary string of implications:  $x \in B \rightarrow x \in (A \cap B)$  and  $x \in (A \cap B) \rightarrow x \in A \wedge x \in B$  so  $x \in B \rightarrow x \in A$  and thus  $B \subseteq A$ .