Give a DFA that accepts the language accepted by the following NFA.


Answer:


Discussion: Let $D$ be the DFA and $N$ be the NFA.
The start state of $D$ corresponds to $\partial^{*}\left(p_{0}, \epsilon\right)$, that is, the set of states of $N$ containing $N$ 's start state and everything reachable from that state via $\epsilon$-transitions. (There are no $\epsilon$-transitions here, so it is just $\left\{p_{0}\right\}$.)

q0

Now, repeatedly find a state $q$ in $D$ whose out-transitions haven't been added, and add them: for each input symbol $a$, look at all of $N$ 's states that can be reached from any one of the $p_{i}$ corresponding to $q$ by consuming $a$ (including any subsequent
$\epsilon$-transitions). Add state $q^{\prime}=\bigcup \partial^{*}\left(p_{i}, a\right)$ if not already present and add transition $\delta(q, a)=q^{\prime}$ to $D$.
$q_{0}$ doesn't have out-transitions yet. From $p_{0}, a$ goes to $p_{0}$, so add a transition $q_{0} \rightarrow q_{0}$ to $D$. From $p_{0}, b$ goes to $p_{0}$ or $p_{1}$, so add a state $\left\{p_{0}, p_{1}\right\}$ and a transition from $q_{0}$ to that state.

$q_{1}$ doesn't have out-transitions yet. From $p_{0}, a$ goes to $p_{0}$ and from $p_{1}, a$ goes to $p_{2}$. From $p_{0}, b$ goes to $p_{0}$ or $p_{1}$ and from $p_{1}, b$ goes to $p_{2}$.

$q_{2}$ doesn't have out-transitions yet. From $p_{0}, a$ goes to $p_{0}$ and from $p_{2}, a$ goes to $p_{3}$. From $p_{0}, b$ goes to $p_{0}$ or $p_{1}$ and from $p_{2}, b$ goes to $p_{3}$.

$q_{3}$ doesn't have out-transitions yet. From $p_{0}, a$ goes to $p_{0}$, from $p_{1}, a$ goes to $p_{2}$, and from $p_{2}, a$ goes to $p_{3}$. From $p_{0}, b$ goes to $p_{0}$ or $p_{1}$, from $p_{1}, b$ goes to $p_{2}$, and from $p_{2}, b$ goes to $p_{3}$.

$q_{4}$ doesn't have out-transitions yet. From $p_{0}, a$ goes to $p_{0}$, and from $p_{3}, a$ goes nowhere. From $p_{0}, b$ goes to $p_{0}$ or $p_{1}$, and from $p_{3}, b$ goes nowhere.

$q_{5}$ doesn't have out-transitions yet. From $p_{0}, a$ goes to $p_{0}$, from $p_{1}, a$ goes to $p_{2}$, and from $p_{3}, a$ goes nowhere. From $p_{0}, b$ goes to $p_{0}$ or $p_{1}$, from $p_{1}, b$ goes to $p_{2}$, and from $p_{3}, b$ goes nowhere.

$q_{6}$ doesn't have out-transitions yet. From $p_{0}, a$ goes to $p_{0}$, from $p_{2}, a$ goes to $p_{3}$, and from $p_{3}, a$ goes nowhere. From $p_{0}, b$ goes to $p_{0}$ or $p_{1}$, from $p_{2}, b$ goes to $p_{3}$, and from $p_{3}, b$ goes nowhere.

$q_{7}$ doesn't have out-transitions yet. From $p_{0}, a$ goes to $p_{0}$, from $p_{1}, a$ goes to $p_{2}$, from $p_{2}, a$ goes to $p_{3}$, and from $p_{3}, a$ goes nowhere. From $p_{0}, b$ goes to $p_{0}$ or $p_{1}$, from $p_{1}, b$ goes to $p_{2}$, from $p_{2}, b$ goes to $p_{3}$, and from $p_{3}, b$ goes nowhere.


The final step is that any state of $D$ containing a final state of $N$ is a final state.

3. Give a DFA that accepts the language accepted by the following NFA. (Be sure to note that, for example, it is possible to reach both $q_{1}$ and $q_{3}$ from $q_{0}$ on consumption of an $a$, because of the $\varepsilon$-transition.)


Answer:


Discussion: Let $D$ be the DFA and $N$ be the NFA.
The start state of $D$ corresponds to $\partial^{*}\left(q_{0}, \epsilon\right)$, that is, the set of states of $N$ containing $N$ 's start state and everything reachable from that state via $\epsilon$-transitions. (There are no $\epsilon$-transitions from $q_{o}$, so it is just $\left\{q_{0}\right\}$.)

p0

Now, repeatedly find a state $p$ in $D$ whose out-transitions haven't been added, and add them: for each input symbol $a$, look at all of $N$ 's states that can be reached from any one of the $q_{i}$ corresponding to $p$ by consuming $a$ (including any subsequent $\epsilon$-transitions). Add state $p^{\prime}=\bigcup \partial^{*}\left(q_{i}, a\right)$ if not already present and add transition $\delta(p, a)=p^{\prime}$ to $D$.
$p_{0}$ doesn't have out-transitions yet. From $q_{0}, a$ goes to $q_{0}$ or $q_{1}$, then $\epsilon$-transitions get to $q_{3}$. From $q_{0}, b$ goes to $q_{2}$, then $\epsilon$-transitions get to $q_{1}$ and $q_{3}$.

$p_{1}$ doesn't have out-transitions yet. From $q_{0}, a$ goes to $q_{0}$ or $q_{1}$, then $\epsilon$-transitions get to $q_{3}$; from $q_{1}, a$ goes nowhere; and from $q_{3}, a$ goes to $q_{3}$. From $q_{0}, b$ goes to $q_{2}$, then $\epsilon$-transitions get to $q_{1}$ and $q_{3}$; from $q_{1}, b$ goes to $q_{1}$, then $\epsilon$-transitions get to $q_{3}$; and from $q_{3}, b$ goes to $q_{4}$.

$p_{2}$ doesn't have out-transitions yet. From $q_{1}, a$ goes nowhere; from $q_{2}, a$ goes to $q_{2}$ or $q_{4}$, then $\epsilon$-transitions get to $q_{1}$ and $q_{3}$; and from $q_{3}, a$ goes to $q_{3}$. From $q_{1}, b$ goes to $q_{1}$, then $\epsilon$-transitions get to $q_{3}$; from $q_{2}, b$ goes nowhere; and from $q_{3}, b$ goes to $q_{4}$.

$p_{3}$ doesn't have out-transitions yet. From $q_{1}, a$ goes nowhere; from $q_{2}, a$ goes to $q_{2}$ or $q_{4}$, then $\epsilon$-transitions get to $q_{1}$ and $q_{3}$; from $q_{3}, a$ goes to $q_{3}$; and from $q_{4}, a$ goes nowhere. From $q_{1}, b$ goes to $q_{1}$, then $\epsilon$-transitions get to $q_{3}$; from $q_{2}, b$ goes nowhere; from $q_{3}, b$ goes to $q_{4}$; and from $q_{4}, b$ goes to $q_{4}$.

$p_{4}$ doesn't have out-transitions yet. From $q_{1}, a$ goes nowhere; from $q_{3}, a$ goes to $q_{3}$; and from $q_{4}$, a goes nowhere. From $q_{1}, b$ goes to $q_{1}$, then $\epsilon$-transitions get to $q_{3}$; from $q_{3}, b$ goes to $q_{4}$; and from $q_{4}, b$ goes to $q_{4}$.

$p_{5}$ doesn't have out-transitions yet. From $q_{3}, a$ goes to $q_{3}$. From $q_{3}, b$ goes to $q_{4}$.

$p_{6}$ doesn't have out-transitions yet. From $q_{4}, a$ goes nowhere. From $q_{4}, b$ goes to $q_{4}$.


The final step is that any state of $D$ containing a final state of $N$ is a final state.


