Give a DFA that accepts the language accepted by the following NFA.



Discussion: Let D be the DFA and N be the NFA.

The start state of D corresponds to  $\partial^*(p_0, \epsilon)$ , that is, the set of states of N containing N's start state and everything reachable from that state via  $\epsilon$ -transitions. (There are no  $\epsilon$ -transitions here, so it is just  $\{p_0\}$ .)



Now, repeatedly find a state q in D whose out-transitions haven't been added, and add them: for each input symbol a, look at all of N's states that can be reached from any one of the  $p_i$  corresponding to q by consuming a (including any subsequent  $\epsilon$ -transitions). Add state  $q' = \bigcup \partial^*(p_i, a)$  if not already present and add transition  $\delta(q, a) = q'$  to D.

 $q_0$  doesn't have out-transitions yet. From  $p_0$ , a goes to  $p_0$ , so add a transition  $q_0 \rightarrow q_0$  to D. From  $p_0$ , b goes to  $p_0$  or  $p_1$ , so add a state  $\{p_0, p_1\}$  and a transition from  $q_0$  to that state.



 $q_1$  doesn't have out-transitions yet. From  $p_0$ , a goes to  $p_0$  and from  $p_1$ , a goes to  $p_2$ . From  $p_0$ , b goes to  $p_0$  or  $p_1$  and from  $p_1$ , b goes to  $p_2$ .



 $q_2$  doesn't have out-transitions yet. From  $p_0$ , a goes to  $p_0$  and from  $p_2$ , a goes to  $p_3$ . From  $p_0$ , b goes to  $p_0$  or  $p_1$  and from  $p_2$ , b goes to  $p_3$ .



 $q_3$  doesn't have out-transitions yet. From  $p_0$ , a goes to  $p_0$ , from  $p_1$ , a goes to  $p_2$ , and from  $p_2$ , a goes to  $p_3$ . From  $p_0$ , b goes to  $p_0$  or  $p_1$ , from  $p_1$ , b goes to  $p_2$ , and from  $p_2$ , b goes to  $p_3$ .



 $q_4$  doesn't have out-transitions yet. From  $p_0$ , a goes to  $p_0$ , and from  $p_3$ , a goes nowhere. From  $p_0$ , b goes to  $p_0$  or  $p_1$ , and from  $p_3$ , b goes nowhere.



 $q_5$  doesn't have out-transitions yet. From  $p_0$ , a goes to  $p_0$ , from  $p_1$ , a goes to  $p_2$ , and from  $p_3$ , a goes nowhere. From  $p_0$ , b goes to  $p_0$  or  $p_1$ , from  $p_1$ , b goes to  $p_2$ , and from  $p_3$ , b goes nowhere.



 $q_6$  doesn't have out-transitions yet. From  $p_0$ , a goes to  $p_0$ , from  $p_2$ , a goes to  $p_3$ , and from  $p_3$ , a goes nowhere. From  $p_0$ , b goes to  $p_0$  or  $p_1$ , from  $p_2$ , b goes to  $p_3$ , and from  $p_3$ , b goes nowhere.



 $q_7$  doesn't have out-transitions yet. From  $p_0$ , a goes to  $p_0$ , from  $p_1$ , a goes to  $p_2$ , from  $p_2$ , a goes to  $p_3$ , and from  $p_3$ , a goes nowhere. From  $p_0$ , b goes to  $p_0$  or  $p_1$ , from  $p_1$ , b goes to  $p_2$ , from  $p_2$ , b goes to  $p_3$ , and from  $p_3$ , b goes nowhere.





The final step is that any state of D containing a final state of N is a final state.

**3.** Give a DFA that accepts the language accepted by the following NFA. (Be sure to note that, for example, it is possible to reach both  $q_1$  and  $q_3$  from  $q_0$  on consumption of an a, because of the  $\varepsilon$ -transition.)



Answer:



Discussion: Let D be the DFA and N be the NFA.

The start state of D corresponds to  $\partial^*(q_0, \epsilon)$ , that is, the set of states of N containing N's start state and everything reachable from that state via  $\epsilon$ -transitions. (There are no  $\epsilon$ -transitions from  $q_o$ , so it is just  $\{q_0\}$ .)



Now, repeatedly find a state p in D whose out-transitions haven't been added, and add them: for each input symbol a, look at all of N's states that can be reached from any one of the  $q_i$  corresponding to p by consuming a (including any subsequent  $\epsilon$ -transitions). Add state  $p' = \bigcup \partial^*(q_i, a)$  if not already present and add transition  $\delta(p, a) = p'$  to D.

 $p_0$  doesn't have out-transitions yet. From  $q_0$ , a goes to  $q_0$  or  $q_1$ , then  $\epsilon$ -transitions get to  $q_3$ . From  $q_0$ , b goes to  $q_2$ , then  $\epsilon$ -transitions get to  $q_1$  and  $q_3$ .



 $p_1$  doesn't have out-transitions yet. From  $q_0$ , a goes to  $q_0$  or  $q_1$ , then  $\epsilon$ -transitions get to  $q_3$ ; from  $q_1$ , a goes nowhere; and from  $q_3$ , a goes to  $q_3$ . From  $q_0$ , b goes to  $q_2$ , then  $\epsilon$ -transitions get to  $q_1$  and  $q_3$ ; from  $q_1$ , b goes to  $q_1$ , then  $\epsilon$ -transitions get to  $q_3$ ; and from  $q_3$ , b goes to  $q_4$ .



 $p_2$  doesn't have out-transitions yet. From  $q_1$ , a goes nowhere; from  $q_2$ , a goes to  $q_2$  or  $q_4$ , then  $\epsilon$ -transitions get to  $q_1$  and  $q_3$ ; and from  $q_3$ , a goes to  $q_3$ . From  $q_1$ , b goes to  $q_1$ , then  $\epsilon$ -transitions get to  $q_3$ ; from  $q_2$ , b goes nowhere; and from  $q_3$ , b goes to  $q_4$ .



 $p_3$  doesn't have out-transitions yet. From  $q_1$ , a goes nowhere; from  $q_2$ , a goes to  $q_2$  or  $q_4$ , then  $\epsilon$ -transitions get to  $q_1$  and  $q_3$ ; from  $q_3$ , a goes to  $q_3$ ; and from  $q_4$ , a goes nowhere. From  $q_1$ , b goes to  $q_1$ , then  $\epsilon$ -transitions get to  $q_3$ ; from  $q_2$ , b goes nowhere; from  $q_3$ , b goes to  $q_4$ ; and from  $q_4$ , b goes to  $q_4$ .



 $p_4$  doesn't have out-transitions yet. From  $q_1$ , a goes nowhere; from  $q_3$ , a goes to  $q_3$ ; and from  $q_4$ , a goes nowhere. From  $q_1$ , b goes to  $q_1$ , then  $\epsilon$ -transitions get to  $q_3$ ; from  $q_3$ , b goes to  $q_4$ ; and from  $q_4$ , b goes to  $q_4$ .



 $p_5$  doesn't have out-transitions yet. From  $q_3$ , a goes to  $q_3$ . From  $q_3$ , b goes to  $q_4$ .



 $p_6$  doesn't have out-transitions yet. From  $q_4$ , a goes nowhere. From  $q_4$ , b goes to  $q_4$ .



The final step is that any state of D containing a final state of N is a final state.

