Identify the context-free language accepted by each of the following pushdown automata.

a)

b)

c)

d)
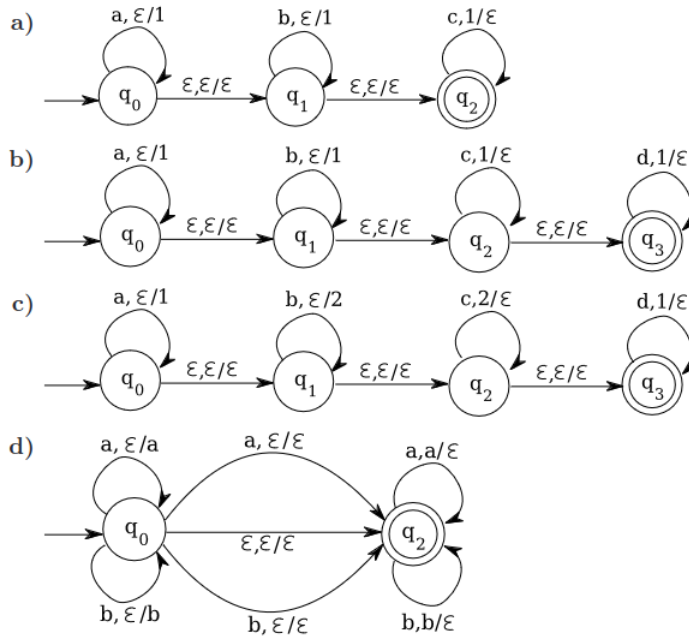
Answer:

(a) A 1 is pushed onto the stack for each a and b, then popped for each c. Since the stack starts empty and must be empty at the end, the number of cs must match the combined total of the as and bs.

$$L = \{\, a^n b^m c^k \mid n + m = k \,\}$$

(b) A 1 is pushed onto the stack for each a and b, then popped for each c and d. Since the stack starts empty and must be empty at the end, the number of cs and ds must match the combined total of the as and bs.

$$L = \{\, a^n b^m c^k d^l \mid n + m = k + l \,\}$$

(c) A 1 is pushed onto the stack for each a and a 2 for each b, then a 2 is popped for each c and 1 for each d. Since the stack starts empty and must be empty at the end, the number of cs and ds must match the number of bs and as, respectively.

$$L = \{\, a^n b^m c^k d^l \mid m = k \wedge n = l \,\}$$

(d) Initially each a and b read is pushed onto the stack, while at the end, those symbols are popped only if the same symbol is read from the string. This requires

a palindrome — the string must end with the same symbols it starts with but in reverse order. In the middle, a single a, a single b, or no symbol at all can be read without any change to the stack — the single character allows for an odd-length string while the ϵ-transition allows for an even-length string.

$$L = \{ \ w\sigma w^R \mid \sigma \in \{a, b, \epsilon\} \ \}$$

Let B be the language over the alphabet $\{(,)\}$ that consists of strings of parentheses that are balanced in the sense that every left parenthesis has a matching right parenthesis. Find a deterministic pushdown automaton with a single state that accepts the language B. Explain how your automaton works, and explain the circumstances in which it will fail to accept a given string of parentheses.
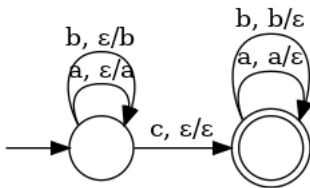
Answer:
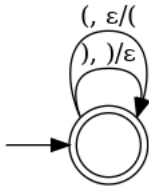
Push (and pop). More (s means the end of the string will be reached with (s still on the stack and more)s means that the stack will be empty before the end of the string is reached, so neither of these will be accepted (only strings with the same number of (and)).

Find a deterministic pushdown automaton that accepts the language

$$\{ \; wcw^R \mid w \in \{a, b\}^* \; \}$$

Answer:

Discussion: This is very similar to one of the examples above — push symbols in the first half of the string, then pop in the second half.