Use the Pumping Lemma to show that $\{xx \mid x \in \{a,b\}^*\}$ is not regular.

Answer: Let N be the threshold length and pick $xx = a^N b^N a^N b^N$. This belongs to our language ($x = a^N b^N$) and $|xx| \geq N$ as required.

Assume that $L = \{xx \mid x \in \{a,b\}^*\}$ is regular; then Theorem 3.6 says that $a^N b^N a^N b^N = xyz$ where $|xy| \leq N$ and $|y| \geq 1$. We observe that xy must consist entirely of as since $a^N b^N a^N b^N$ starts with N as. Let $1 \leq k \leq N$ be the number of as in y, then we can write xyz as $a^{N-k} a^k b^N a^N b^N$ where $y = a^k$. But then $xz = a^{N-k} b^N a^N b^N$, which does not have the form xx. So the assumption that L was regular must be false.

Discussion: Use the same strategy used for $\{a^n b^n \mid n \geq 0\}$: let N be the threshold length, then pick a string of the desired form that is long enough to ensure that xy can only cover one of the symbols. Then xz omits some of those symbobls, disrupting the balance between the two halves of the string.

Use the Pumping Lemma to show that $\{x \mid n_a(x) = n_b(x)\}$ is not regular.

Answer: Let N be the threshold length and pick $x = a^N b^N$. This belongs to our language ($n_a(x) = n_b(x) = N$) and $|x| \geq N$ as required.

Assume that $L = \{x \mid n_a(x) = n_b(x)\}$ is regular; then Theorem 3.6 says that $a^N b^N = xyz$ where $|xy| \leq N$ and $|y| \geq 1$. We observe that xy must consist entirely of as since $a^N b^N$ starts with N as. Let $1 \leq k \leq N$ be the number of as in y, then we can write xyz as $a^{N-k} a^k b^N$ where $y = a^k$. But then $xz = a^{N-k} b^N$, where the number of as and bs are not the same. So the assumption that L was regular must be false.

Discussion: Use the same strategy used for $\{a^n b^n \mid n \geq 0\}$: let N be the threshold length, then pick a string of the desired form that is long enough to ensure that xy can only cover one of the symbols. Then xz omits some of those symbobls, disrupting the balance between the two halves of the string.