

## Shortest Weighted Path



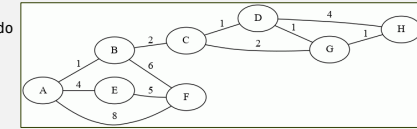
BFS gives the shortest path in terms of the number of edges in the path

What if you have edge weights, and want the shortest path in terms of the sum of the edge weights along the path?

## Shortest Weighted Path

```

bfs(G,s)
for each vertex u in V-{s} do
  state[u] = "undiscovered"
  prev[u] = null
  dist[u] = ∞
state[s] = "discovered"
prev[s] = null
dist[s] = 0
Q.enqueue(s)
while Q is not empty do
  u = Q.dequeue()
  for each edge (u,v) in G.incidentEdges(u) do
    if state[v] = "undiscovered" then
      state[v] = "discovered"
      prev[v] = u
      dist[v] = dist[u] + w(u,v)
      Q.enqueue(v)
  state[u] = "processed"
    
```



A	0		
B	∞	1	
C	∞	∞	3
D	∞	∞	
E	∞	4	
F	∞	8	*
G	∞	∞	
H	∞	∞	

- BFS sets  $dist[v]$  when a vertex is first discovered
- this label will be wrong if there is a longer path (more edges) with a lower cost

## Shortest Weighted Path

Fix.

- allow  $dist[u]$  to be updated each time an edge  $(v,u)$  is encountered
- ensure that vertices along the shortest path  $s \rightarrow u$  are processed before  $u$ 
  - so that  $dist[u]$  won't need to be updated after  $u$  is processed

Implementation.

- observation:  $dist[v] < dist[u]$  for all vertices  $v$  on shortest path  $s \rightarrow u$ 
  - thus handling 'discovered' vertices in order of increasing  $dist[v]$  would satisfy the second part of the fix
  - note: assumes edge weights  $> 0$

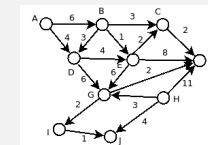
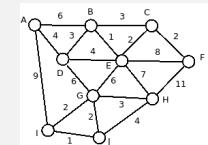
## Dijkstra's Algorithm

finds the shortest path from  $s$  to every other vertex in the graph

to just find the shortest path from  $s$  to  $t$ , exit loop after  $t$  is removed from the PQ

```

algorithm dijkstra(G,s):
for all v in V do
  dist[v] ← ∞
  prev[v] = null
dist[s] ← 0
PQ ← makeQueue(V)
while PQ is not empty do
  v ← PQ.removeMin()
  for each incident edge e=(v,u)
    if dist[u] > dist[v]+w(v,u) then
      dist[u] = dist[v]+w(v,u)
      PQ.decreaseKey(u)
      prev[u] = v
    
```



- can augment algorithm to store shortest path in addition to distance (add prev labels)

## Edsger W Dijkstra



- Dutch computer scientist
- 1930 – 2002
  
- received the 1972 Turing Award for fundamental contributions to the development of structured programming languages
  - characteristics
    - three control structures: ordered sequence of statements, conditionals, loops
    - subroutines
    - blocks
  - in contrast to languages like BASIC which utilized GOTO statements