More Graph Algorithms

Electricity is finally coming to a (very) rural small town. Given the cost of stringing wire between every pair of houses in the town, determine how to connect the town with the smallest possible total cost.

Each connection in a computer network has a maximum bandwidth. Find the maximum bandwidth that can be achieved between computer A to computer B.

How vulnerable is an electrical grid? Determine how many transmission lines would need to be interrupted in order to cut off at least one community from the power plant.

Find the optimal route for plowing all of the roads in a city.

connected components
minimum bottleneck spanning tree
maximum flow
topological sort
postman tour
minimum cut
minimum cost flow
bipartite matching
minimum product spanning tree
shortest path
euler path or cycle
transitive closure
maximum spanning tree
minimum spanning tree
articulation vertex or edge

flow network – edge weights denote max capacity of the edge

minimum bottleneck spanning tree – spanning tree with minimum max edge weight

max flow – max flow that can be sent from s to t while respecting max capacities

postman tour – min-length cycle traversing every edge at least once

min cost flow – cheapest way to send a given flow volume from s to t

bipartite matching – subset of edges such that no two edges share a vertex

minimum product spanning tree – spanning tree that minimizes the product of the edge weights

euler cycle (tour) – cycle traversing every edge exactly once

min cut – cheapest set of edges whose removal partitions the graph into two disjoint subsets

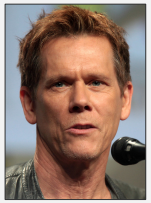
An *edit step* is a transformation from one word x to another word y such that x and y are words in the dictionary, and x can be transformed to y by adding, deleting, or changing one letter. So the transformation from *dig* to *dog* or from *dog* to *do* are both edit steps. An *edit step ladder* is a lexicographically ordered sequence of words $w_1, w_2, \ldots, w_n$ such that the transformation from w_i to w_{i+1} is an edit step for all i from 1 to $n-1$.

For a given dictionary, you are to compute the length of the longest edit step ladder.

In this problem you are given N colorful cubes each having a distinct weight. Each face of a cube is colored with one color. Your job is to build a tower using the cubes you have subject to the following restrictions:

- Never put a heavier cube on a lighter one.
- The bottom face of every cube (except the bottom cube, which is lying on the floor) must have the same color as the top face of the cube below it.
- Construct the tallest tower possible.

Six Degrees of Kevin Bacon and Bacon Numbers



Six degrees of Kevin Bacon: find a way to link a particular actor to Kevin Bacon through movies they have in common.

The **Bacon number** of an actor is the number of degrees of separation he or she has from Bacon, as defined by the game. This is an application of the Erdős number concept to the Hollywood movie industry. The higher the Bacon number, the greater the separation from Kevin Bacon the actor is.

- Kevin Bacon himself has a Bacon number of 0.
- Those actors who have worked directly with Kevin Bacon have a Bacon number of 1.
- If the lowest Bacon number of any actor with whom X has appeared in any movie is N, X's Bacon number is N+1.

Examples [edit]

Elvis Presley:

- Elvis Presley was in *Change of Habit* (1969) with Edward Asner
- Edward Asner was in *JFK* (1991) with Kevin Bacon

Therefore, Asner has a Bacon number of 1, and Presley (who never appeared in a film with Bacon) has a Bacon number of 2.

Ian McKellen:

- Ian McKellen was in *X-Men: Days of Future Past* (2014) with Michael Fassbender and James McAvoy
- McAvoy and Fassbender were in *X-Men: First Class* (2011) with Kevin Bacon

Therefore, McAvoy and Fassbender have Bacon numbers of 1, and McKellen has a Bacon number of 2.

7-15. *[5]* You are planning the seating arrangement for a wedding given a list of guests, V. For each guest g you have a list of all other guests who are on bad terms with them. Feelings are reciprocal: if h is on bad terms with g, then g is on bad terms with h. Your goal is to arrange the seating such that no pair of guests sitting at the same table are on bad terms with each other. There will be only two tables at the wedding. Give an efficient algorithm to find an acceptable seating arrangement if one exists.

7-28. *[5]* Your job is to arrange n ill-behaved children in a straight line, facing front. You are given a list of m statements of the form "i hates j." If i hates j, then you do not want to put i somewhere behind j, because then i is capable of throwing something at j.

(a) Give an algorithm that orders the line (or says that it is not possible) in $O(m+n)$ time.

7-29. *[3]* A particular academic program has n required courses, certain pairs of which have prerequisite relations so that (x, y) means you must take course x before y. How would you analyze the prerequisite pairs to make sure it is possible for people to complete the program?

7-22. *[5]* Consider a set of movies $M_1, M_2, \ldots, M_k$. There is a set of customers, each one of which indicates the two movies they would like to see this weekend. Movies are shown on Saturday evening and Sunday evening. Multiple movies may be screened at the same time.

You must decide which movies should be televised on Saturday and which on Sunday, so that every customer gets to see the two movies they desire. Is there a schedule where each movie is shown at most once? Design an efficient algorithm to find such a schedule if one exists.