

The secret to using the sums table is to arrange the function being summed in the format $f(i) = b^{ai} i^d \log^e i$ and determine b^a , d , and e .

The following table outlines the few easy rules with which you will be able to compute $\Theta(\sum_{i=1}^n f(i))$ for functions with the basic form $f(n) = \Theta(b^{an} \cdot n^d \cdot \log^e n)$. (We consider more general functions at the end of this section.)

b^a	d	e	Type of Sum	$\sum_{i=1}^n f(i)$	Examples
> 1	Any	Any	Geometric Increase (dominated by last term)	$\Theta(f(n))$	$\sum_{i=0}^n 2^{2^i} \approx 1 \cdot 2^{2^n}$ $\sum_{i=0}^n b^i = \Theta(b^n)$ $\sum_{i=0}^n 2^i = \Theta(2^n)$
$= 1$	> -1	Any	Arithmetic-like (half of terms approximately equal)	$\Theta(n \cdot f(n))$	$\sum_{i=1}^n i^d = \Theta(n \cdot n^d) = \Theta(n^{d+1})$ $\sum_{i=1}^n i^2 = \Theta(n \cdot n^2) = \Theta(n^3)$ $\sum_{i=1}^n i = \Theta(n \cdot n) = \Theta(n^2)$ $\sum_{i=1}^n 1 = \Theta(n \cdot 1) = \Theta(n)$ $\sum_{i=1}^n \frac{1}{i^{0.99}} = \Theta(n \cdot \frac{1}{n^{0.99}}) = \Theta(n^{0.01})$
	$= -1$	$= 0$	Harmonic	$\Theta(\ln n)$	$\sum_{i=1}^n \frac{1}{i} = \log_e(n) + \Theta(1)$
	< -1	Any	Bounded tail (dominated by first term)	$\Theta(1)$	$\sum_{i=1}^n \frac{1}{i^{1.001}} = \Theta(1)$ $\sum_{i=1}^n \frac{1}{i^2} = \Theta(1)$
< 1	Any	Any			$\sum_{i=1}^n (\frac{1}{2})^i = \Theta(1)$ $\sum_{i=0}^n b^{-i} = \Theta(1)$

(table from Jeff Edmonds, *How to Think About Algorithms*)

For example –

$$\sum_1^n i = \sum_1^n 1 \cdot 1^i \cdot i^1 \cdot \log^0(i) \quad \text{thus } b^a = 1, d = 1, e = 0$$

$$\sum_1^n 4i^2 = \sum_1^n 4 \cdot 1^i \cdot i^2 \cdot \log^0(i) \quad \text{thus } b^a = 1, d = 2, e = 0$$

$$\sum_1^n 2^{3i} \log^2(i) = \sum_1^n 1 \cdot (2^3)^i \cdot i^0 \cdot \log^2(i) \quad \text{thus } b^a = 8, d = 0, e = 2$$

Also keep in mind that when the 5th column in the table (showing the solution) references n , the “ n ” refers to the upper end of the sum – the column is showing the result for the sum from 1 to n . If you have a sum with a different upper range, it can be less confusing to first rewrite the pattern from the table with another symbol.

For example, $\sum_1^{n^2} i$ is of the $\Theta(n \cdot f(n))$ pattern, but the upper bound of the sum is n^2 rather than n . So, rewrite the table pattern with a new symbol (such as s):

$$\sum_1^s i = \Theta(s \cdot f(s)) \quad . \quad \text{Then substitute } n^2 \text{ for } s \text{ to get the answer: } \sum_1^{n^2} i = \Theta(n^2 \cdot f(n^2)) = \Theta(n^3) \quad .$$