Prerequisites for Math 130

The material below represents only some of the basic material with which you should be familiar. We will not be reviewing this material. You may wish to consult Appendix A in your text as well as parts of Chapter 1.

1. Exponents and radicals, including:
   a) \( x^{m}x^{n} = x^{m+n} \)
   b) \( (x^{m})^{n} = x^{mn} \)
   c) \( \sqrt[n]{x} = x^{1/n} \)
   d) \( \frac{1}{x^n} = x^{-n} \)
   e) \( \sqrt[n]{x^m} = x^{m/n} \)

2. The quadratic formula: If \( ax^2 + bx + c = 0 \) then \( x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \).

3. Interval notation (note the different types of brackets):
   a) Open interval: \( a < x < b \) or \( (a,b) \).
   b) Closed interval: \( a \leq x \leq b \) or \( [a,b] \).
   c) Half-open interval: \( a < x < b \) or \( [a,b) \) and \( a < x \leq b \) or \( (a,b] \).
   d) Various rays: \( x < a \) or \( (-\infty,a) \); \( a \geq x \) or \( [a,\infty) \).
   e) All reals: \( (-\infty,\infty) \).

4. Absolute value: If \( a > 0 \), then:
   a) \( |x| = a \) means \( x = \pm a \).
   b) \( |x| < a \) means \( -a < x < a \) or \( (-a,a) \).
   c) \( |x| \geq a \) means \( -a \leq x \leq a \) or \( [-a,a] \).
   d) \( |x| > a \) means \( x < -a \) or \( x > a \) or \( (-\infty,-a) \cup (a,\infty) \).
   e) \( |x| \leq a \) means \( x \leq a \) or \( x \geq a \) or \( (-\infty,-a] \cup [a,\infty) \).
   f) Note that \( \sqrt{x^2} = |x| \) not just \( x \). (Try a negative value for \( x \) to see why.)

5. a) The expression \( |x-a| \) represents the distance between \( x \) and \( a \). So for example, \( |x-2| = 3 \) says that the distance between \( x \) and 2 is 3. (So \( x \) is either 5 or –1.) You can also use the the expressions above to solve this:

\[
|x-2| = 3 \implies \begin{cases} x-2 = 3 \quad \text{or} \quad x = 5, \\ x-2 = -3 \quad \text{or} \quad x = -1 \end{cases}
\]

b) With an inequality such as \( |x-2| < 3 \), again use the basic definitions. \( |x-2| < 3 \) means \( -3 < (x-2) < 3 \) or \( -1 < x < 5 \).

6. The distance formula for the distance between two points \( (x_1,y_1) \) and \( (x_2,y_2) \) is \( \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} \). This can be derived directly from the Pythagorean theorem if you draw the two points and a right triangle determined by them.

7. Equations of lines:
   a) Slope-intercept form: \( y = mx + b \).
   b) Point-slope form: \( y - y_0 = m(x-x_0) \), where \( (x_0,y_0) \) is a point on the line. This is particularly useful for calculus.
   c) Know how to obtain the equation of line from two points \( (x_1,y_1) \) and \( (x_2,y_2) \).

8. Functions including domain and range. Composition of functions \( f \circ g(x) = f(g(x)) \). For example, if \( f(x) = x^2 - 6 \) and \( g(x) = 1 + 2x^3 \), then

\[
f \circ g(x) = f(g(x)) = f(1 + 2x^3) = (1 + 2x^3)^2 - 6 = 1 + 4x^3 + 4x^6 - 6 = 4x^3 + 4x^6 - 5.
\]
9. Basic geometry formulas:
   a) Triangles, Area: \( A = \frac{1}{2}bh \).
   b) Rectangles, Area: \( A = lw \), Perimeter: \( P = 2l + 2w \).
   c) Circles, Area: \( A = \pi r^2 \), Circumference: \( C = 2\pi r \).
   d) Spheres, Volume: \( V = \frac{4}{3}\pi r^3 \), Surface Area: \( SA = 4\pi r^2 \).
   e) Cylinder, Volume: \( V = \pi r^2 h \), Surface Area: \( SA = 2\pi r^2 + 2\pi rh \).
   f) Cone, Volume: \( V = \frac{1}{3}\pi r^2 h \).
   g) Rectangular box, Volume: \( V = lwh \), Surface Area: \( SA = 2lw + 2lh + 2wh \).

10. We will always measure angles in radians. The conversion factors are:
    a) \( \pi \) radians = 180°.
    b) So \( 1^\circ = \frac{\pi}{180} \) rad
    c) and 1 rad = \( \frac{180^\circ}{\pi} \).

11. Recall for a right triangle like the one below, we can define the basic trig functions in terms of the sides of the triangle.

```
    hyp
   /   \
 /     \
 θ     \ opp
     /     
     adj
```

   a) \( \sin \theta = \frac{\text{opp}}{\text{hyp}} \)  
   b) \( \cos \theta = \frac{\text{adj}}{\text{hyp}} \)  
   c) \( \tan \theta = \frac{\text{opp}}{\text{adj}} \)  
   d) \( \sec \theta = \frac{\text{hyp}}{\text{adj}} \)

12. For more general angles,

```
    r
   /|
 / |
\θ / | \( (x,y) \)
   |
   |
   |
   x
```

   a) \( \sin \theta = \frac{y}{r} \)  
   b) \( \cos \theta = \frac{x}{r} \)  
   c) \( \tan \theta = \frac{y}{x} \)  
   d) \( \sec \theta = \frac{r}{x} \)

13. You should know the values of the trig functions at these basic angles.

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Elementary Practice

Here's a quick review of even more basic topics. I assume that you know all of this material extremely well.

Exponents and radicals:

1. Simplify and evaluate: \((2\sqrt{5})(2\sqrt{5})(5\sqrt{2})(5\sqrt{2})\)

2. Write in simplified exponential form:
   a) \((x^3)^2\)  
   b) \((x^3 \cdot y^2)^{-1}\)
   c) \((-2x)^{-5}\)
   d) \(\frac{x \cdot y^{-2}}{y^{-4}x^{-3}}\)
   e) \((-2)^0\)
   f) \((z^{-2})^{-2}\)

3. Simplify each expression:
   a) \((-8z^6)^{2/3}\)
   b) \(6y^2(2y^4)^2\)
   c) \(\frac{25y^8}{10y^4}\)
   d) \(\frac{12(x + y)^3}{9(x + y)^5}\)
   e) \(\frac{10(x - y)^{5/3}}{15(x - y)^{9/3}}\)

4. Rewrite each expression below in simplified exponential form (no radicals):
   a) \(\frac{1}{\sqrt{x}}\)
   b) \(\sqrt[3]{x^6}\)
   c) \(6\sqrt[3]{y^2}\)
   d) \(2\sqrt[3]{z}\)
   e) \(\frac{1}{2\sqrt[3]{z^3}}\)
   f) \(\sqrt[6]{(x + 1)^3}\)

5. Expand each of these powers:
   a) \((x - 2)^3\)
   b) \((x - \sqrt{5})^2\)

6. Factor each expression completely:
   a) \(x^2 + 10x + 25\)
   b) \(4x^2 - 4xy + y^2\)
   c) \(t^2 - t - 6\)
   d) \(49 - 9y^2\)
   e) \(9u^2 - 4v^2\)
   f) \(6x^2 - 54\)
   g) \(-2x^3 + 2x^2 + 4x\)

7. Simplify these rational functions.
   a) \(\frac{8(x + 1)^4 + 7x^3(x + 1)^2}{(x + 1)^4}\)
   b) \(\frac{9(x - 2)^3(x + 1) - 2(x - 2)^4(x - 1)^2}{(x - 2)^6}\)

Composition

8. Let \(f(x) = x^2 - 4x + 1\), \(g(x) = 2 - x\), and \(h(x) = 3\sin x\), determine the expressions for
   a) \(f \circ g(x)\)
   b) \(h \circ g(x)\)
Exponents and radicals:

1. \((2\sqrt{5})(2\sqrt{5})(5\sqrt{2})(5\sqrt{2}) = (4 \times 5)(25 \times 2) = 1000\)

2. Simplified expressions:
   
   a) \(x^6\)  
   b) \(x^{-3}y^{-2}\)  
   c) \(-32x^{-5}\)  
   d) \(x^2y^2\)  
   e) 1  
   f) \(z^4\)

3. Simplified:
   
   a) \(4z^4\)  
   b) \(24y^{10}\)  
   c) \(\frac{5y^4}{2}\)  
   d) \(\frac{4(x + y)^{-2}}{3}\)  
   e) \(\frac{2(x - y)^{-4/3}}{3}\)

4. Exponential form:
   
   a) \(\frac{1}{\sqrt{x}} = x^{-1/2}\)  
   b) \(\sqrt[6]{x^6} = x^{2/3}\)  
   c) \(6\sqrt[5]{y^2} = 6y^{2/5}\)  
   d) \(2\sqrt[3]{z} = 2z^{1/3}\)  
   e) \(\frac{1}{2\sqrt[3]{z^3}} = \frac{z^{-4/3}}{2}\)  
   f) \((x + 1)^{4/6} = (x + 1)^{2/3}\)

5. Expanded powers:
   
   a) \(x^3 - 6x^2 + 12x - 8\)  
   b) \(x^2 - 2x\sqrt{5} + 5\)

6. Factored:
   
   a) \((x + 5)^2\)  
   b) \((2x - y)^2\)  
   c) \((t - 3)(t + 2)\)  
   d) \((7 - 3y)(7 + 3y)\)  
   e) \((3u - 2v)(3u + 2v)\)  
   f) \(6(x - 3)(x + 3)\)  
   g) \(-2x(x - 2)(x + 1)\)

7. Simplified:
   
   a) \(\frac{8(x + 1)^2 + 7x^3}{(x + 1)^2}\)  
   b) \(\frac{9(x + 1) - 2(x - 2)(x - 1)^2}{(x - 2)^3}\)

8. Composition: Let \(f(x) = x^2 - 4x + 1\), \(g(x) = 2 - x\), and \(h(x) = 3\sin x\), determine the expressions for
   
   a) \(f(g(x)) = f(2 - x) = (2 - x)^2 - 4(2 - x) + 1 = x^2 - 3\)
   
   b) \(h(g(x)) = h(2 - x) = 3\sin(2 - x)\)
Math 130 Prereqs Hand In

Review the material on the previous pages as needed. You may wish to consult Appendix A and Chapter 1 of your text.

1. Factor completely and simplify: \((x + 3)^2 - 4(x + 3)\).

2. Notation and Piecewise Functions. (See page 12, Examples 3 and 4.) Define

\[
f(x) = \begin{cases} 
  x + 2 & \text{if } x < 1, \\
  x^2 + 1 & \text{if } x \geq 1.
\end{cases}
\]

Evaluate the following:

a) \(f(3) = \)

b) \(f(-6) = \)

c) \(f(1) = \)

d) \(f(0.99) = \)

e) \(f(f(0)) = \)

f) Draw a graph of \(f(x)\) on the axes below.

3. Page 8 #28

4. Page 8 #42

5. Page 9 #54

   A) 
   B) 
   C) 

6. Page 19 #16

   \[
f(x) = \begin{cases} 
  \text{Work, if needed.}
\end{cases}
\]

7. a) According to page 29,

   \[
y = \log_b x \text{ provided } \]

   b) Now do page 33 #34.

The next problems are in the text. Show your work.
8. Give the exact value (no decimals) for each of these trig and inverse trig functions.
   a) \( \sin \frac{\pi}{3} \)
   b) \( \cos \frac{\pi}{2} \)
   c) \( \tan \frac{\pi}{3} \)
   d) \( \sin^{-1} \frac{1}{2} \)
   e) \( \cos^{-1} \frac{1}{2} \)

9. Complete the table.

<table>
<thead>
<tr>
<th>Interval Notation</th>
<th>Set Notation</th>
<th>Graph</th>
</tr>
</thead>
<tbody>
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<td>((-\infty, -4])</td>
<td>( {x; 3 \leq x \leq \frac{11}{2}} )</td>
<td>![Graph Image]</td>
</tr>
<tr>
<td>((-1, 7))</td>
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</table>

10. a) According to “Properties of the Absolute Value” in Appendix A page 1063: \( |x| < a \) means the same thing as

b) Use the property above to solve \( |2x - 4| < 6 \).

11. Extra Credit
   a) Page 20 #26 (see Example 6 in the text).

   b) (You will need to use a calculator.)

   **Height of a Mountain** In traveling across flat land, you notice a mountain directly in front of you. Its angle of elevation (to the peak) is 3.5°. After you drive 13 miles closer to the mountain, the angle of elevation is 9°. Approximate the height of the mountain.

   ![Height of a Mountain Diagram]
Background Information

1. Name (and nick name): ____________________________ Section 8:00 AM

2. Year: FY, SO, JR, SR College: HC, WSC

3. Phone number:

4. Advisor:

5. (Potential) Major(s): Minor(s):

6. I am taking this course because

7. What was your favorite course (any subject) at HWS (and/or high school if you are a first-year student)? Why was this course your favorite?

8. Career interests and aspirations:

9. Hobbies, sports, or other activities that you enjoy (other than doing math!):

10. I have read the syllabus and understand both the grading and the attendance policies. I understand that missing more than three classes and/or labs will lower my grade and that missing more than six may lead to automatic expulsion from the course.

   Signature: ________________________________