Math 130 Day 02 Hand In At Class. Name: Answers

0. Do WeBWorKSet Day02. Do them with your homework below. They are similar. Today's class and Lab from Thursday should help you to get started on them. Also, see the online class notes for Day 2.

1. Review: Factoring. Simplify each expression by canceling any common factors. See WeBWorK Set Day02, #1.

a) $\dfrac{x^2-7x+10}{x-5}$ for $x \neq 5$

$$\frac{\cancel{(x-5)}(x-2)}{\cancel{x-5}} = x-2 \quad (x \neq 5)$$

b) $\dfrac{4x^2-20x-24}{x-6}$ for $x \neq 6$

$$\frac{4(x^2-5x-6)}{x-6} = \frac{4(x+1)\cancel{(x-6)}}{\cancel{x-6}} = 4(x+1)$$

2. If $f(x) = 2x^2+2x+2$, simplify the difference quotient $\dfrac{f(x)-f(1)}{x-1}$ (where $x \neq 1$). WeBWorK Set Day02, #2.

$$\frac{f(x)-f(1)}{x-1} = \frac{2x^2+2x+2-\overset{f(2)}{6}}{x-1} = \frac{2x^2+2x-4}{x-1} = \frac{2(x^2+x-2)}{x-1}$$

$$= \frac{2\cancel{(x-1)}(x+2)}{\cancel{x-1}} = 2(x+2) \quad (x \neq 1)$$

3. Page 59, #9. These are just basic calculations. Nothing fancy. Write out the average velocity and then use a calculator to compute the result.

a) $[1,4]$

$$\frac{s(4)-s(1)}{4-1} = \frac{-16(4^2)+128(4)-\overset{112}{[-16(1)^2+128(1)]}}{} = \frac{-256+512-112}{3} = \boxed{48 \text{ m/s}}$$

b) $[1,3]$

$$\frac{s(3)-s(1)}{3-1} = \frac{-16(3)^2+128(3)-112}{2} = \frac{-144+384-112}{2} = \boxed{64 \text{ m/s}}$$

c) $[1,2]$

$$\frac{s(2)-s(1)}{2-1} = \frac{-16(2)^2+128(2)-112}{1} = \frac{-64+256-112}{1} = \boxed{80 \text{ m/s}}$$

d) $[1,1+h]$. Your answer will be a formula that contains h.

$$\frac{s(1+h)-s(1)}{1+h-1} = \frac{-16(1+h)^2+128(1+h)-112}{h} = \frac{-16(1+2h+h^2)+128+128h-112}{h}$$

$$= \frac{\cancel{-16}-32h-16h^2+\cancel{128}+128h-\cancel{112}}{h} = \frac{-16h^2+96h}{h} = \boxed{-16h+96 \text{ m/s}}$$

4. Average and Instantaneous Velocity. The position of an object is give by the function $f(x) = x^2 + 6x$.

a) Find and simplify the expression for average velocity on the interval $[1, x]$. This is the same as $m_{\text{sec}} = \dfrac{f(x) - f(1)}{x - 1}$ (where $x \neq 1$).

$$\frac{f(x)-f(1)}{x-1} = \frac{(x^2+6x)-(1+6)}{x-1} = \frac{x^2+6x-7}{x-1} = \frac{(\cancel{x-1})(x+7)}{\cancel{x-1}}$$

$$= x+7 \quad (x \neq 1)$$

b) Fill in the table below with the average velocities (as we did in the in class examples).

$[1, x]$	m_{sec} = ave vel on $[1, x]$
$[1, 2]$	$\cancel{x}+7 = 2+7 = 9$
$[1, 1.5]$	$1.5+7 = 8.5$
$[1, 1.1]$	8.1
$[1, 1.01]$	8.01
$[1, 1.001]$	8.001

c) Make a conjecture (educated guess) about the instantaneous velocity right at $x = 1$. $\boxed{8 \text{ m/s}}$

5. Average and Instantaneous Velocity. The position of an object is give by the function $f(x) = \ln x$. The expression for average velocity on the interval $[1, x]$ is

$$\text{average velocity} = m_{\text{sec}} = \frac{f(x) - f(0)}{x - 0} = \frac{\ln x - \ln 1}{x - 1}, \text{ where } x \neq 0.$$

There is not much simplification that you can do other than to evaluate $\ln 1$. So this time you will need to use a calculator to evaluate the following average velocities. (Make sure it is set to **radians**.)

$$\frac{\ln x - \ln 1}{x-1} = \frac{\ln x - 0}{x-1} = \frac{\ln x}{x-1}$$

$[1, x]$	m_{sec} = Ave Vel on $[1, x]$	
$[1, 1.1]$	$\ln 1.1 / .1 \approx 0$	$.9531$
$[1, 1.01]$	$\ln .01 / .01 \approx$	$.99503$
$[1, 1.001]$	$\ln .001 / .001 \approx$	$.99950$

Make a conjecture (educated guess) about the instantaneous velocity right at $x = 0$. 1.0

6. a) Finding the general formula for a secant slope. (See WeBWorK Set Day02, #2.) Let $f(x) = x^2 + x + 3$. The point $P = (4, 23)$ lies on the curve. Suppose that $Q = (x, x^2 + x + 3)$ is any other point on the graph of f. Find the slope of the secant line, m_{sec} through P and Q. Simplify your answer. It will be a function of x.

$$m_{sec} = \frac{f(x) - f(4)}{x-4} = \frac{\overset{f(x)}{x^2+x+3} - \overset{f(4)}{23}}{x-4} = \frac{x^2+x-20}{x-4}$$

$$= \frac{(x+5)\cancel{(x-4)}}{\cancel{x-4}}$$

$$= x+5 \qquad (x \neq 4)$$

b) The algebra is a little harder in this one. Let $f(x) = \frac{1}{x}$. The point $P = (5, \frac{1}{5})$ lies on the curve. Let $Q = (x, \frac{1}{x})$ be any other point on the graph of f. Find the slope of the secant line, m_{sec}, through P and Q. Simplify your answer. It will be a function of x.

$$\frac{f(x) - f(5)}{x-5} = \frac{\frac{1}{x} - \frac{1}{5}}{x-5} = \frac{\frac{5-x}{5x}}{x-5} = \frac{5-x}{5x(x-5)}$$

$$= \frac{-1\cancel{(x-5)}}{5x\cancel{(x-5)}}$$

$$= -\frac{1}{5x} \qquad (x \neq 5)$$

7. Five-Minute Review: Polynomial Functions (See Section 1.2, page 12). A **polynomial** is a function of the form

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$$

where n is a non-negative integer and the a's are real numbers. If $a_n \neq 0$, then n is the **degree** of the polynomial (largest power). Note: All the powers of x must be non-negative integers. No fractional powers, no negative powers. Which of the following are polynomials? For those that are polynomials, what is their **degree**?

a) $-\frac{2}{3}x^5 + 3x^4 + x^2 - 11$ Deg 5 b) $5x^2 - x^{1/3} - 23$ c) 4 Deg 0 d) $6x^{-2} + 4x^{-1} + 2$

e) $\sqrt{3x^{12} + 11x^9 + 12}$ f) $\frac{2x^2 + x}{7x + 1}$ g) $\frac{1}{6x^2} + \frac{2}{x}$ h) $6^{1/2}x^3 - 4x + 7$ Deg 3

i) Look up the definition of a **rational function** on page 13. Then find a rational function in the list above.

Rational (f) or (g)