

Math 130 Day 6: Hand in Monday. Name: Answers

WeBWorK Set Day 06 due Monday night. Do it early. It has both 'regular' and 'infinite' limits.

1. Warm-up. Determine the following as we did in class; not all are infinite limits!!

a) $\lim_{x \rightarrow 3^+} \frac{\overbrace{x^2}^{\rightarrow \infty}}{\underbrace{(x-3)^3}_{\rightarrow (0^+)^3 = 0^+}} = +\infty$ and $\lim_{x \rightarrow 3^-} \frac{\overbrace{x^2}^{\rightarrow \infty}}{\underbrace{(x-3)^3}_{\rightarrow (0^-)^3 = 0^-}} = -\infty$ so $\lim_{x \rightarrow 3} \frac{x^2}{(x-3)^3}$ DNE

b) $\lim_{x \rightarrow 3^+} \frac{\overbrace{x^2}^{\rightarrow \infty}}{\underbrace{(x-3)^2}_{\rightarrow (0^+)^2 = 0^+}} = +\infty$ and $\lim_{x \rightarrow 3^-} \frac{\overbrace{x^2}^{\rightarrow \infty}}{\underbrace{(x-3)^2}_{\rightarrow (0^-)^2 = 0^+}} = +\infty$ so $\lim_{x \rightarrow 3} \frac{x^2}{(x-3)^2} + \infty$

- c) Hint: Factor, then determine what the numerator and denominator approach

$$\lim_{x \rightarrow -2^+} \frac{\overbrace{x-1}^{\rightarrow -1}}{\underbrace{x^2-x-6}_{\rightarrow 0^+ (-5) = 0^-}} = \lim_{x \rightarrow -2^+} \frac{x-1}{(x+2)(x-3)} = +\infty$$

- d) Hint: Factor.

$$\lim_{x \rightarrow -2^+} \frac{\overbrace{x^2+2x}^{\rightarrow 0}}{\underbrace{x^2-x-6}_{\rightarrow 0^+ (-5) = 0^-}} = \lim_{x \rightarrow -2^+} \frac{x(x+2)}{(x+2)(x-3)} = \lim_{x \rightarrow -2^+} \frac{x}{x-3} \stackrel{\text{Rat'l}}{\rightarrow} -5 = -2/-5 = \frac{2}{5}$$

2. Let $f(x) = \begin{cases} 2mx^2 + 3, & \text{if } x < -1 \\ 9, & \text{if } x = -1 \\ \frac{m^2 x^2}{2x+3}, & \text{if } x \geq -1 \end{cases}$. Determine all values for m for which $\lim_{x \rightarrow -1} f(x)$ exists. Show your work which should include limit calculations. (See Lab 2 #5 for a similar problem, with answers online.)

We need $\lim_{x \rightarrow -1^+} f(x) = \lim_{x \rightarrow -1^-} f(x)$ for $\lim_{x \rightarrow -1} f(x)$ to exist

so $\lim_{x \rightarrow -1^-} f(x) = \lim_{x \rightarrow -1^-} 2m(x^2)^1 + 3 = 2m + 3$ } Must be equal

$$\lim_{x \rightarrow -1^+} f(x) = \lim_{x \rightarrow -1^+} \frac{m^2 x^2}{2x+3} = \frac{m^2}{-2+3} = m^2$$

So: $m^2 = 2m + 3$ or $m^2 - 2m - 3 = (m-3)(m+1) = 0$

$$\boxed{m=3 \text{ or } -1}$$

3. Let $f(x) = 10 + 2x - x^2$. Evaluate $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$. (See Lab 2 #8 for a similar problem, with answers online.)

$$\begin{aligned}\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} &= \lim_{h \rightarrow 0} \frac{(10 + 2(x+h) - (x+h)^2) - [10 + 2x - x^2]}{h} \\&= \lim_{h \rightarrow 0} \frac{10 + 2x + 2h - x^2 - 2xh - h^2 - 10 - 2x + x^2}{h} \\&= \lim_{h \rightarrow 0} \frac{2h - 2xh - h^2}{h} \\&= \lim_{h \rightarrow 0} 2 - 2x - h \\&= 2 - 2x\end{aligned}$$

Math 130 Day 6: Hand in Wednesday. Name: Answers

WeBWorK Set Day07 due Thursday night. (Yes, it is called Day07 and this is only Day 6. Don't ask... Do it early. It has both 'regular' and 'infinite' limits.

1. Here are five straightforward one-sided limits problems. Use $+\infty$ or $-\infty$ if appropriate. When the denominator (bottom) goes to 0 but the numerator does not, determine the signs of each, then determine the limit. For indeterminate " $\frac{0}{0}$ " limits do more work. Indicate your work or reasoning for each solution.

a) $\lim_{x \rightarrow 3^+} \frac{-2x + 1}{x - 3} = -\infty$ 

$\nearrow -5$
 $\searrow 0^+$

b) $\lim_{x \rightarrow -4^-} \frac{x^2 + 3x - 4}{x + 4} = \lim_{x \rightarrow -4^-} \frac{(x+4)(x-1)}{x+4} = \lim_{x \rightarrow -4^-} x-1 = -5$
poly
no VA

$\nearrow 0$
 $\searrow 0$

c) $\lim_{x \rightarrow -2^+} \frac{x^2 - 4}{x + 4} = \frac{0}{2} = 0$ 
rational
no VA

$\nearrow 0$
 $\searrow 2$

d) $\lim_{x \rightarrow 1^+} \frac{x^2}{x^2 - 1} = +\infty$ 
 $\searrow 0^+$

e) $\lim_{x \rightarrow 2^-} \frac{x^2 + 1}{x(x-2)} = -\infty$ 
 $\nearrow 5$
 $\searrow 2(0^-) = 0^-$

2. Beside each of the functions in problem 1 (all five parts) write "VA" if the function has a vertical asymptote at the point x is approaching. In one sentence how can you tell from your work if there is a VA?

* The function has a VA at $x = a$ if either $\lim_{x \rightarrow a^+} f(x) = \pm \infty$
or $\lim_{x \rightarrow a^-} f(x) = \pm \infty$

3. a) Does $f(x) = \frac{2x-8}{x^2(x-4)}$ have a VA at $x = 0$? Explain carefully using limits. Use a limit from either side of 0. E.g.

$$\lim_{x \rightarrow 0^+} \frac{2x-8}{x^2(x-4)} \stackrel{x \rightarrow 0^+}{=} +\infty \text{ . since } \lim_{x \rightarrow 0^+} f(x) = +\infty, \text{ there's a VA @ } x=0$$

$\hookrightarrow 0^+(-4) = 0^-$

b) Does $f(x) = \frac{2x-8}{x^2(x-4)}$ have a VA at $x = 4$? Explain carefully using limits. Again use a one-sided limit.

$$\lim_{x \rightarrow 4^+} \frac{2x-8}{x^2(x-4)} \stackrel{x \rightarrow 4^+}{=} \lim_{x \rightarrow 4^+} \frac{2(x-4)}{x^2(x-4)} = \lim_{x \rightarrow 4^+} \frac{2}{x^2} \stackrel{\text{Rat'l}}{=} \frac{2}{16} = \frac{1}{8} \neq \infty$$

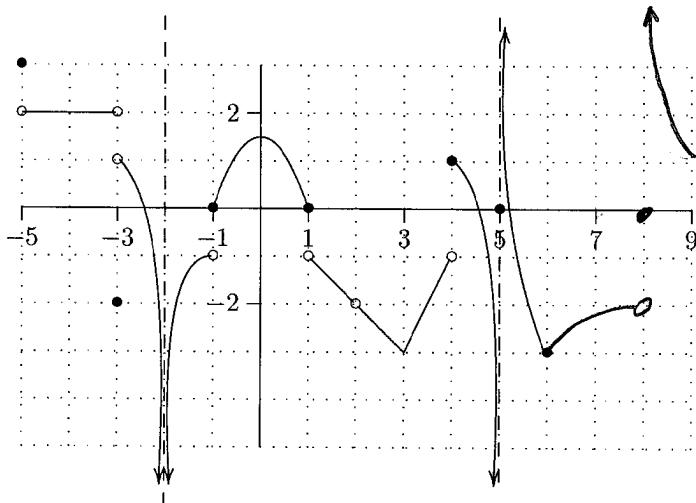
$\hookrightarrow 0$

we get the same limit as $x \rightarrow 4^-$

$$\lim_{x \rightarrow 4^-} \frac{2x-8}{x^2(x-4)} \stackrel{x \rightarrow 4^-}{=} \lim_{x \rightarrow 4^-} \frac{2(x-4)}{x^2(x-4)} \stackrel{\text{Rational}}{=} \frac{1}{8} \neq \infty$$

There is no VA at $x = 4$.

4. a) Use the graph of f to evaluate each of the expressions in the table or explain why the value does not exist. Note: Use $+\infty$ and $-\infty$ when appropriate.



a	$\lim_{x \rightarrow a^-} f(x)$	$\lim_{x \rightarrow a^+} f(x)$	$\lim_{x \rightarrow a} f(x)$	$f(a)$
-4	2	2	2	2
-3	2	1	DNE	-2
-2	$-\infty$	$-\infty$	$-\infty$	UND
-1	-1	0	DNE	0
1	0	-1	DNE	0
2	-2	-2	-2	UND
3	-3	-3	-3	-3
4	-1	1	DNE	1
5	$-\infty$	∞	DNE	0

- b) Complete the graph of the function above for $x = 6$ to 9 so that all of the following are true:

$$\lim_{x \rightarrow 8^-} f(x) = -2, \quad \lim_{x \rightarrow 8^+} f(x) = +\infty, \quad f(8) = 0.$$