Math 130, Day ~~38~~ 7 Hand-In: Name: _____

You should be able to determine the limits of each of these functions very easily. Note where you use "highest powers." Use $+\infty$ and $-\infty$ where appropriate.

1. a) $\lim_{x\to+\infty} -2x^5+3x^3-1 \overset{HP}{=} \lim_{x\to+\infty} -2\overset{\to+\infty}{x^5} = -\infty$

b) $\lim_{x\to-\infty} 3x-2x^2 \overset{HP}{=} \lim_{x\to-\infty} -2\overset{\to+\infty}{x^2} = -\infty$

c) $\lim_{x\to-\infty} 4x^3+19x^2+1 \overset{HP}{=} \lim_{x\to-\infty} 4\overset{\to-\infty}{x^3} = -\infty$

2. a) $\lim_{x\to+\infty} \frac{3-2x}{3x^3-1} \overset{HP}{=} \lim_{x\to+\infty} \frac{-2x}{3x^3} = \lim_{x\to\infty} \frac{\overset{\to-2}{-2}}{\underset{\to\infty}{x^2}} = 0$

b) $\lim_{x\to+\infty} \frac{3-2x}{3x-1} \overset{HP}{=} \lim_{x\to+\infty} \frac{-2x}{3x} = \lim_{x\to+\infty} -\frac{2}{3} = -\frac{2}{3}$

c) $\lim_{x\to-\infty} \frac{3-2x^2}{3x-1} \overset{HP}{=} \lim_{x\to-\infty} \frac{-2x^2}{3x} = \lim_{x\to-\infty} \frac{-2\overset{\to-\infty}{x}}{3} = +\infty$

3. a) $\lim_{x\to+\infty} \frac{5x^{3/2}}{4x^2+1} \overset{HP}{=} \lim_{x\to+\infty} \frac{5x^{3/2}}{4x^2} = \lim_{x\to+\infty} \frac{5}{4\underset{\to\infty}{x^{1/2}}} = 0$

b) $\lim_{x\to+\infty} \frac{5x^{3/2}}{4x^{3/2}+x} \overset{HP}{=} \lim_{x\to+\infty} \frac{5x^{3/2}}{4x^{3/2}} = 5/4$

4. a) Does $f(x) = \dfrac{6x^2 - 12x}{x^2 - 4}$ have any horizontal asymptotes? Justify your answer with appropriate limit calculations.

Check $\lim_{x \to +\infty} f(x)$ and $\lim_{x \to -\infty} f(x)$

1) $\lim_{x \to \infty} \dfrac{6x^2 - 12x}{x^2 - 4} \overset{HP}{=} \lim_{x \to \infty} \dfrac{6x^2}{x^2} = 6$ so $y = 6$ is HA

2) $\lim_{x \to -\infty} \dfrac{6x^2 - 12x}{x^2 - 4} \overset{HP}{=} \lim_{x \to -\infty} \dfrac{6x^2}{x^2} = 6$ (Again $y = 6$ is HA)

b) Where does f have vertical asymptotes? Justify your answer with appropriate limit calculations. Where should you check?

Check where $f(x) = \dfrac{6x^2 - 12x}{x^2 - 4} = \dfrac{6x(x-2)}{(x-2)(x+2)}$ is not defined, $(x = 2, -2)$

Use 1-sided limits!

At $x = -2$

$$\lim_{x \to -2^+} \frac{6x\cancel{(x-2)}}{\cancel{(x-2)}(x+2)} = \lim_{x \to -2^+} \frac{\overset{-12}{6x}}{\underset{\to 0^+}{x+2}} = -\infty \text{ ; so } \boxed{x = -2 \text{ is VA}}$$

At $x = +2$

$$\lim_{x \to +2^+} \frac{6x\cancel{(x-2)}}{\cancel{(x-2)}(x+2)} = \lim_{x \to +2^+} \frac{6x}{x+2} \overset{\text{Rat'l}}{=} \frac{12}{4} = 3 \text{ ,}$$

and $\lim_{x \to 2^-} \dfrac{6x(x-2)}{(x-2)(x+2)} = \lim_{x \to 2^-} \dfrac{6x}{x+2} = 3$

Check both sides (or use 2-sided limit), so No VA @ $x = 2$

c) Bonus: Does $f(x)$ have a **removable discontinuity**? Justify your answer with appropriate limit calculations. Remember $x = a$ is a removable discontinuity if $\lim_{x \to a} f(x)$ exists BUT is NOT EQUAL to $f(a)$.

Yes, at $x = 2$, $\lim_{x \to 2} f(x) = 3$, but $f(2)$ DNE

They are not equal, so there's an RD there