Lab Ticket: Hand In at Lab Tomorrow. ANSWERS

1. This is a good problem to see if you understand the concepts we have been studying. Fill in the table using the information given. For some, several correct answers are possible.

a	$\lim\limits_{x\to a^-} f(x)$	$\lim\limits_{x\to a^+} f(x)$	$\lim\limits_{x\to a} f(x)$	$f(a)$	Left Cont	Right Cont	Cont	RD	VA
-1	0	0	0	DNE	No	No	No	Yes	No
1	4	1	DNE	4	Yes	No	No	No	No
2	2	2	2	Not 2	No	No	No	Yes	No
3	3	3	3	Not 3	No	No	No	Yes	No
4	∞ or $-\infty$	1	DNE	1	No	Yes	No	No	Yes
5	$-\infty$	$-\infty$	$-\infty$	Not $\pm\infty$	No	No	No	No	Yes
6	1	1	1	1	Yes	Yes	Cont	No	No

2. Evaluate these limits. A variety of techniques is required. Use $+\infty$ or $-\infty$, if appropriate.

a) $$\lim_{x\to 0^-} \frac{x-1}{x^2(x+8)} = \lim_{x\to 0^-} \frac{\overbrace{x-1}^{-1}}{\underbrace{x^2(x+8)}_{(0^+)^2\cdot 8=0^+}} = -\infty$$

b) $$\lim_{x\to 1^+} \frac{x-2}{1-\sqrt{x}} = \lim_{x\to 1^+} \frac{\overbrace{x-2}^{-1}}{\underbrace{1-\sqrt{x}}_{0^-}} = +\infty$$

c) $$\lim_{x\to -\infty} \frac{3x-2}{\sqrt{4x^2+1}} \overset{\text{HP}}{=} \lim_{x\to -\infty} \frac{3x}{\sqrt{4x^2}} \lim_{x\to -\infty} \frac{3x}{|2x|} \overset{x\leq 0}{=} \lim_{x\to -\infty} \frac{3x}{-2x} = -\frac{3}{2}$$

☞ Extra Credit

3. Bonus: Like a Quiz/Test Question.

a) Carefully explain where $f(x) = \dfrac{x^2+5x+6}{x^2+2x-3}$ is NOT continuous. Hint: What type of function is this?

Factor:

$$f(x) = \frac{x^2+5x+6}{x^2+2x-3} = \frac{(x+2)(x+3)}{(x+3)(x-1)}.$$

f is rational; so it is continuous except at $x = -3$ and 1 where it is undefined.

b) Using limits determine where $f(x)$ has (1) vertical asymptotes, and (2) removable discontinuities. [Where should you look.] Use appropriate limits to justify each. See the definitions on p. 1.

At $x = -3$:

$$\lim_{x \to -3} f(x) = \lim_{x \to -3} \frac{(x+2)(x+3)}{(x+3)(x-1)} = \lim_{x \to -3} \frac{x+2}{x-1} = \frac{-1}{-4} = \frac{1}{4}.$$

So $x = -3$ is an RD since $\lim\limits_{x \to -3} f(x)$ exists but does not equal $f(-3)$ which DNE.

However $x = 1$ is a VA because checking the one-sided limits:

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} \frac{(x+2)(x+3)}{(x+3)(x-1)} = \lim_{x \to 1^+} \frac{\overbrace{x+2}^{3}}{\underbrace{x-1}_{0^+}} = +\infty$$

OR

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} \frac{(x+2)(x+3)}{(x+3)(x-1)} = \lim_{x \to 1^-} \frac{\overbrace{x+2}^{3}}{\underbrace{x-1}_{0^-}} = +\infty$$

In either case, f has a VA at $x = 1$ since a one-sided limit is infinite there.

c) *Check your understanding.* Give an equation of a rational function with a VA at $x = -2$ and a removable discontinuity at $x = 6$. (Hint: Look back at what happened in the first parts of this problem to create RD's and VA's.)

Use a rational function such as $f(x) = \frac{x-6}{(x+2)(x-6)}$.