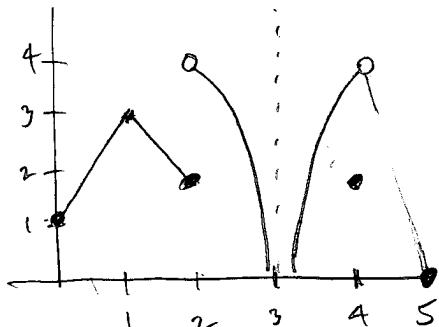


Math 130, Day 9. Hand in Next Class. Name: Answers

1. a) Page 108 #12. Give the FIRST reason from the continuity checklist that fails for each of the points where  $f(x)$  is not continuous.



Continuity Checklist

①  $f(a)$  exists

②  $\lim_{x \rightarrow a} f(x)$  exists (finite)

③  $\lim_{x \rightarrow a} f(x) = f(a)$

Not Continuous at

$x=2$ :  $\lim_{x \rightarrow 2} f(x)$  DNE

$x=3$ :  $f(3)$  DNE

$x=4$ :  $f(4) = 2 \checkmark$

$\lim_{x \rightarrow 4} f(x) = 4 \checkmark$

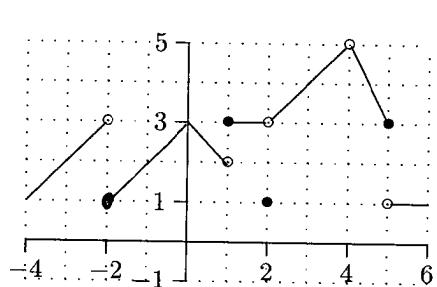
But  $\lim_{x \rightarrow 4} f(x) \neq f(4)$

$$\text{left continuous } \lim_{x \rightarrow 2^-} f(x) = f(2)$$

- b) Now do page 109 #38. (See the answers to #37 for a similar problem.) Your answer should be a set of intervals starting at 0 and ending at 5.

Right continuous  $[0, 2]$   $\cup (2, 3) \cup (3, 4) \cup (4, 5]$       left continuous      left continuous

2. a) Use the graph and the definitions to fill in the table.



| $a$ | $\lim_{x \rightarrow a^-} f(x)$ | $\lim_{x \rightarrow a^+} f(x)$ | $\lim_{x \rightarrow a} f(x)$ | $f(a)$ | Left Cont | Rt Cont | Cont | RD |
|-----|---------------------------------|---------------------------------|-------------------------------|--------|-----------|---------|------|----|
| 1   | 2                               | 3                               | DNE                           | 3      | N         | Y       | N    | N  |
| 2   | 3                               | 3                               | 3                             | 1      | N         | N       | N    | Y  |
| 3   | 4                               | 4                               | 4                             | 4      | Y         | Y       | Y    | N  |
| 4   | 5                               | 5                               | 5                             | DNE    | N         | N       | N    | Y  |
| 8   | 2                               | 2                               | 2                             | 2      | Y         | Y       | Yes  | N  |

- b) Complete the graph so that  $f$  is continuous from the RIGHT at  $x = -2$  but NOT from the LEFT.

3. Page 108 #18. Use the continuity checklist.

$$f(x) = \begin{cases} \frac{x^2-4x+3}{x-3} & \text{if } x \neq 3 \\ 2 & \text{if } x=3 \end{cases}$$

1)  $f(3) = 2 \checkmark$

2)  $\lim_{x \rightarrow 3} f(x) = \lim_{x \rightarrow 3} \frac{x^2-4x+3}{x-3} \stackrel{\cancel{(x-3)}}{\rightarrow} 0 = \lim_{x \rightarrow 3} \frac{(x-3)(x-1)}{x-3} = \lim_{x \rightarrow 3} x-1 = 2 \checkmark$

3)  $\lim_{x \rightarrow 3} f(x) = 2 = f(3) \checkmark$

So  $f$  is continuous @  $x=3$

4. Page 109 #24. Use the theorem the text suggests. (There should be no limit calculations.) Give your answer as a set of intervals.

$S(x) = \frac{x^2 - 4x + 3}{x^2 - 1}$  is rational, so it is continuous except

where the denominator = 0.  $x^2 - 1 = (x-1)(x+1) = 0$

Not continuous at  $x = 1, -1$

Intervals:  $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

5. Page 109 #40.  $f(x) = \begin{cases} x^3 + 4x + 1 & x \leq 0 \\ 2x^3 & x > 0 \end{cases}$

a) Show f not continuous @  $x=0$ . use checklist

first  $\rightarrow$  1.  $f(0) \stackrel{x \leq 0}{=} 0^3 + 4(0) + 1 = 1 \checkmark$  (defined)

next  $\rightarrow$  2. To determine  $\lim_{x \rightarrow 0} f(x)$ , use 1-sided limits since

$f(x)$  is a piecewise function "split" at  $x=0$

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} x^3 + 4x + 1 \stackrel{\text{poly}}{=} 1 \quad \left. \begin{array}{l} \text{Explain briefly} \\ \text{different} \end{array} \right\}$$

and  $\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} 2x^3 \stackrel{\text{poly}}{=} 0 \quad \left. \begin{array}{l} \text{so } \lim_{x \rightarrow 0} f(x) \text{ DNE} \\ \text{at } x=0 \end{array} \right\}$

Explain

$\rightarrow$  condition 2 fails, therefore f is not continuous at  $x=0$

b) Since  $\lim_{x \rightarrow 0^-} f(x) = 1 = f(0)$ , f is continuous from the left at 0.

Explain  
why left cont

and  $\rightarrow (\lim_{x \rightarrow 0^+} f(x) = 0 \neq f(0))$ , so not continuous from right at 0)  
not right cont.  $\rightarrow$   $0 \neq 1$

c) Intervals of continuity

$$(-\infty, 0] \cup (0, \infty)$$

$\uparrow$  include when left continuous

6. Evaluate these limits. Be careful. Use proper limit grammar.

$$a) \lim_{x \rightarrow 2^+} \frac{x+1}{x(2-x)^3} = -\infty$$

$\hookrightarrow 2 \cdot (0^+)^3 = 0^+$

$$b) \lim_{x \rightarrow 2} \frac{x^2 - 3x + 2}{x^2 - 2x} = \lim_{x \rightarrow 2} \frac{(x-2)(x-1)}{x(x-2)} = \lim_{x \rightarrow 2} \frac{x-1}{x} = \frac{1}{2}$$

$\nearrow 0$  more work  
 $\searrow 0$  Rat' l

- c) Is the function above continuous at  $x = 2$ ? Or does it have an RD or a VA? Explain using the appropriate definition.

Not continuous @ 2 b/c  $f(2)$  is not defined.

RD @  $x=2$  b/c  $\lim_{x \rightarrow 2} f(x)$  exists and does not equal  $f(2)$

$$d) \lim_{x \rightarrow 2^-} \frac{3x}{\sqrt{x} - \sqrt{2}} \text{ No VA @ } x=2. \quad \lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^-} f(x) = 1/2, \text{ not } \pm \infty$$

$$\lim_{x \rightarrow 2^-} \frac{3x}{\sqrt{x} - \sqrt{2}} = -\infty$$

$\nearrow 0$   
 $\searrow 0^-$

Do not multiply out

$$e) \lim_{x \rightarrow 1} \frac{x-1}{\sqrt{2x} - \sqrt{x+1}} \cdot \frac{\sqrt{2x} + \sqrt{x+1}}{\sqrt{2x} + \sqrt{x+1}} = \lim_{x \rightarrow 1} \frac{(x-1)(\sqrt{2x} + \sqrt{x+1})}{2x - (x+1)} \leftarrow \text{ simplify}$$

$$= \lim_{x \rightarrow 1} \frac{(x-1)(\sqrt{2x} + \sqrt{x+1})}{x-1} = \lim_{x \rightarrow 1} \sqrt{2x} + \sqrt{x+1} \stackrel{\text{Erat power}}{\leftarrow} \sqrt{2} + \sqrt{2} = 2\sqrt{2}$$

7. Remember to use the definitions and appropriate limits. Let  $f(x) = \begin{cases} x^2 + ax + 1, & \text{if } x < 3 \\ b, & \text{if } x = 3 \\ \frac{x^2 - 3x}{x - 3}, & \text{if } x > 3 \end{cases}$

a) Determine a value of  $b$  that makes  $f(x)$  RIGHT continuous at  $x = 3$ . Explain.

We need  $\lim_{x \rightarrow 3^+} f(x) = f(3) = b$ , so

$$\lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^+} \frac{x^2 - 3x}{x - 3} = \lim_{x \rightarrow 3^+} \frac{x(x-3)}{x-3} = \lim_{x \rightarrow 3^+} x = 3$$

$$\text{So we need } \boxed{3 = b}$$

"Explanation"

b) Now that you know the value of  $b$ , determine a value of  $a$  that makes  $f(x)$  LEFT continuous at  $x = 3$ . Explain.

We need  $\lim_{x \rightarrow 3^-} f(x) = f(3) = b = 3$

$$\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow -} x^2 + ax + 1 \stackrel{\text{poly}}{\rightarrow} 9 + 3a + 1 = 10 + 3a$$

$$\text{We need } 10 + 3a = 3 = f(3)$$

so

$$3a = -7$$

$$\boxed{a = -\frac{7}{3}}$$